

The Gabor filter

```
reset():  
t0:=time():
```

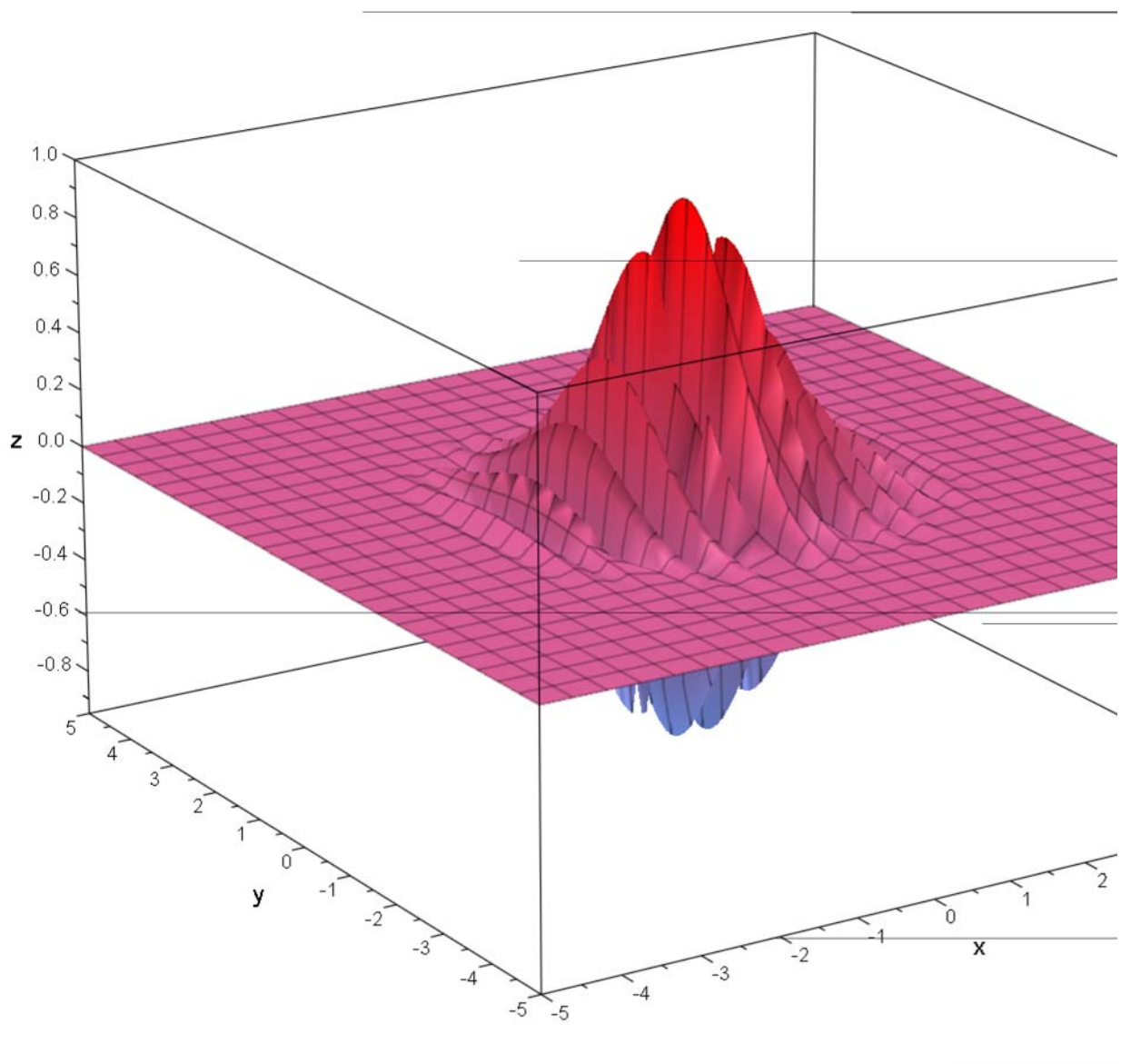
```
myplot:=g->plotfunc3d(args(), AdaptiveMesh=2, Height=20*unit::cm, Width=  
gaborfilter:=(sx,sy,f)->fp::unapply(exp(-(x^2/sx^2+y^2/sy^2)/2)*cos(2*P:  
g:=gaborfilter(1,1,1);
```

$$(x, y) \rightarrow \cos(2 \cdot \pi \cdot x) \cdot e^{-\frac{x^2}{2} - \frac{y^2}{2}}$$

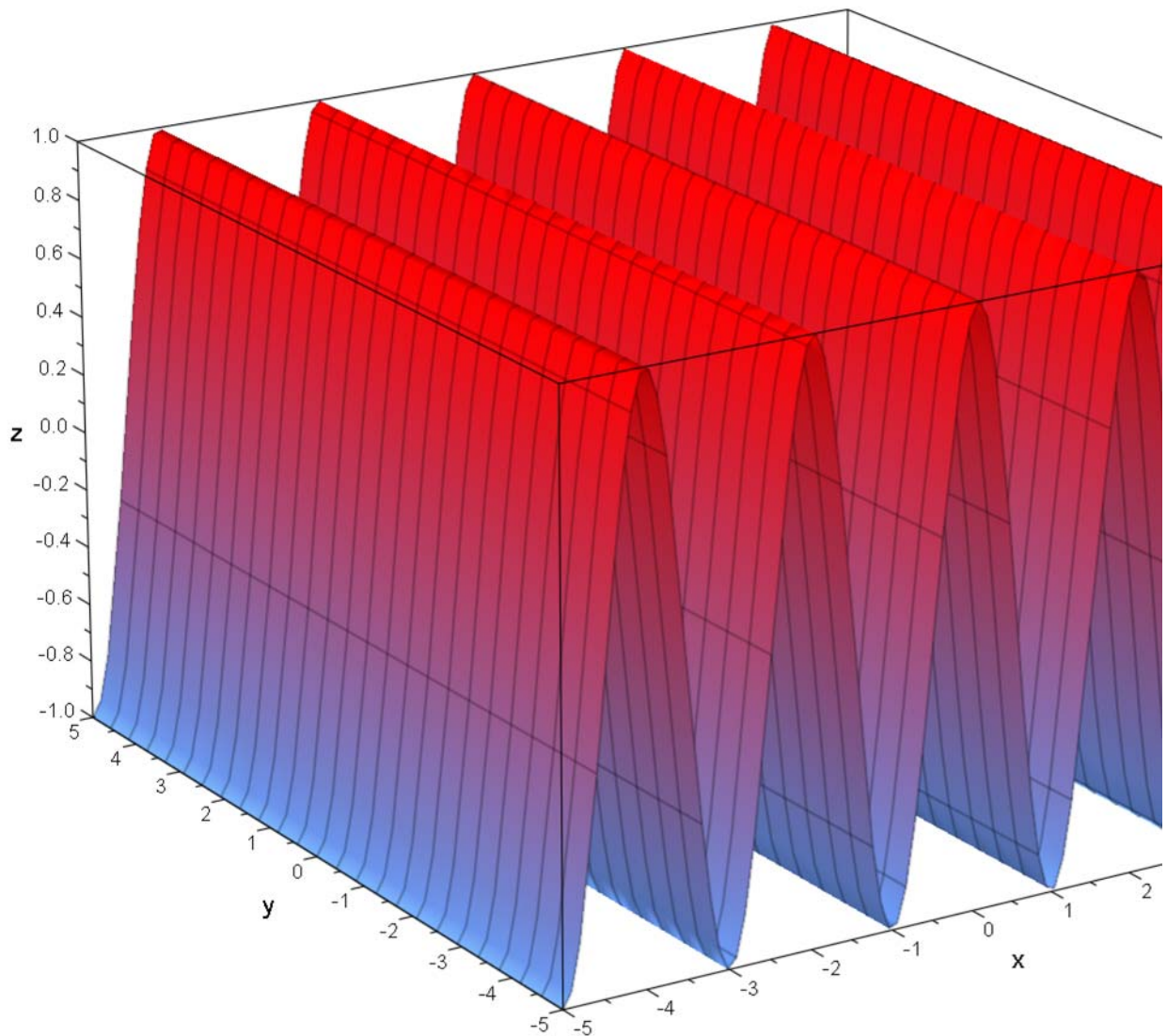
```
int( Simplify( int( gaborfilter(a,b,f)(x,y), y=-infinity..infinity ) ), x=-infin  
assuming a>0 assuming b>0 assuming f>0:  
Igaborfilter:=fp::unapply(% , a, b, f);
```

$$(a, b, f) \rightarrow 2 \cdot \pi \cdot a \cdot b \cdot e^{-2 \cdot \pi^2 \cdot a^2 \cdot f^2}$$

```
myplot(gaborfilter(1,1,f),x=-5..5,y=-5..5,f=0..2);
```



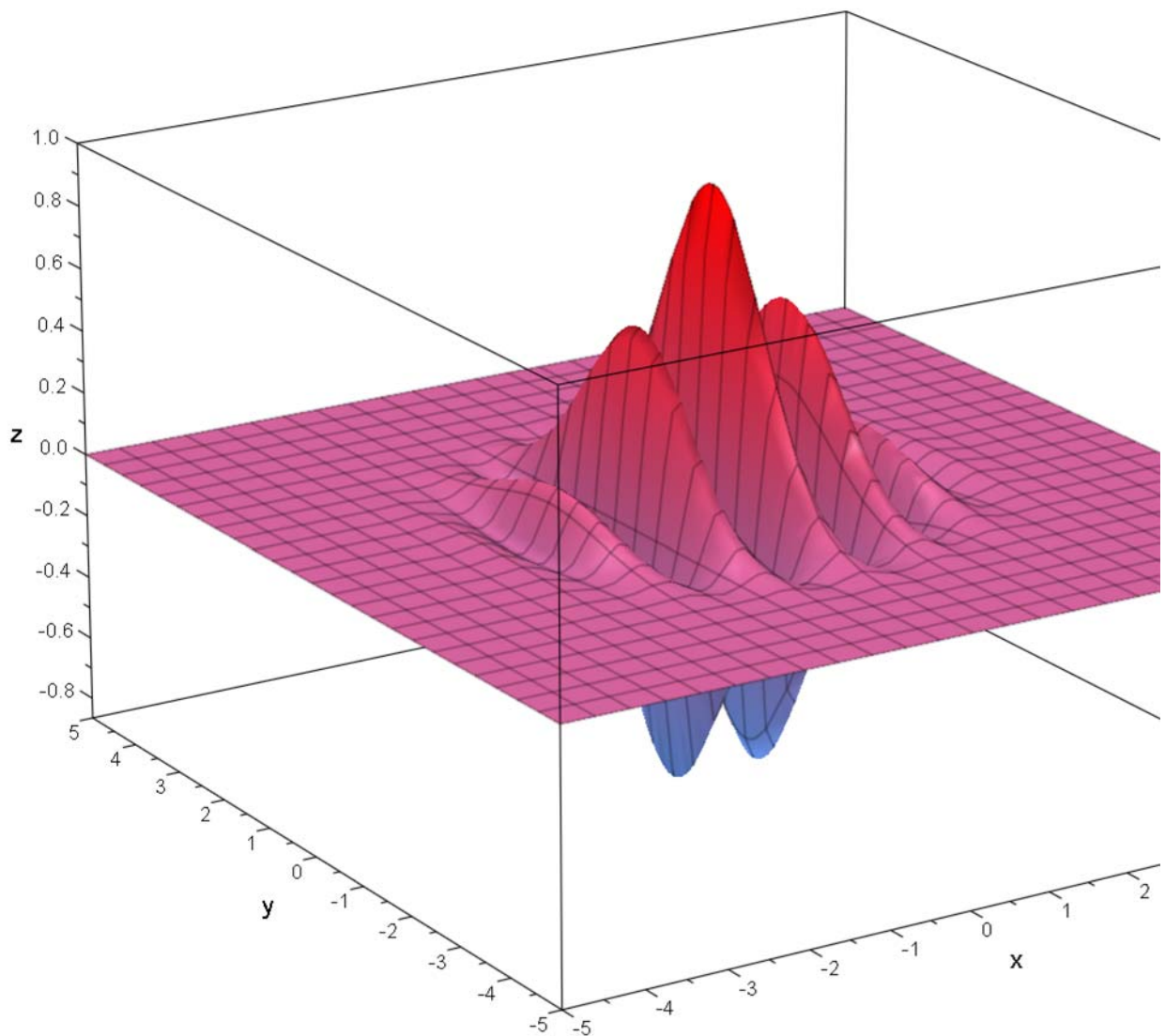
```
// Abstand  
s1:=gaborfilter(infinity,infinity,1/2);  
myplot(s1);  
(x, y) → cos(π · x)
```



```
hold(int)( hold(int)( simplify( Simplify((g*s1)(x,y)), sin ), x=-infinite
```

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\frac{\cos(\pi \cdot x) \cdot e^{-\frac{x^2}{2} - \frac{y^2}{2}}}{2} + \frac{\cos(3 \cdot \pi \cdot x) \cdot e^{-\frac{x^2}{2} - \frac{y^2}{2}}}{2} \right) dx dy = 0.02259396792$$

```
myplot( simplify( simplify(gaborfilter(infinity,infinity,f)(x,y)*g(x,y),sin) ),
```



```

sf:=gaborfilter(infinity,infinity,f);
kf:=simplify( Simplify((g*%)(x,y)), sin );
k2:=1/2*(gaborfilter(1,1,1-f)(x,y)+gaborfilter(1,1,1+f)(x,y));
simplify(kf-k2);
hold(int)( hold(int)( kf, x=-infinity..infinity), y=-infinity..infinity);
= (vf:=1/2*(Igaborfilter(1,1,1-f)+Igaborfilter(1,1,1+f)));

```

$(x, y) \rightarrow \cos(2 \cdot \pi \cdot f \cdot x)$

$$e^{-\frac{x^2}{2} - \frac{y^2}{2}} \cdot \frac{\cos(2 \cdot \pi \cdot x - 2 \cdot \pi \cdot f \cdot x)}{2} + e^{-\frac{x^2}{2} - \frac{y^2}{2}} \cdot \frac{\cos(2 \cdot \pi \cdot x + 2 \cdot \pi \cdot f \cdot x)}{2}$$

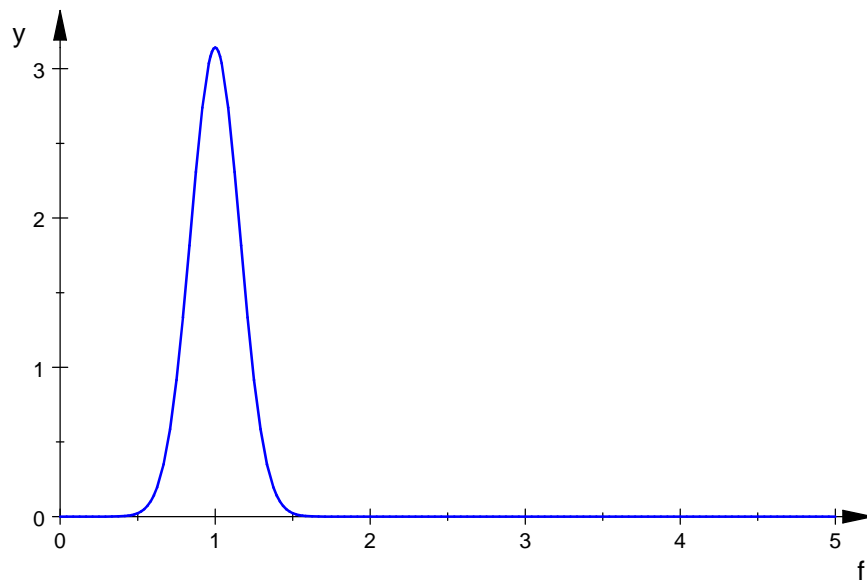
$$\frac{\cos(2 \cdot \pi \cdot x \cdot (f - 1)) \cdot e^{-\frac{x^2}{2} - \frac{y^2}{2}}}{2} + \frac{\cos(2 \cdot \pi \cdot x \cdot (f + 1)) \cdot e^{-\frac{x^2}{2} - \frac{y^2}{2}}}{2}$$

$$\frac{\cos(2 \cdot \pi \cdot x \cdot (f-1)) \cdot e^{-\frac{x^2}{2} - \frac{y^2}{2}}}{2} + \frac{\cos(2 \cdot \pi \cdot x \cdot (f+1)) \cdot e^{-\frac{x^2}{2} - \frac{y^2}{2}}}{2}$$

0

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\frac{e^{-\frac{x^2}{2} - \frac{y^2}{2}} \cdot \cos(2 \cdot \pi \cdot x - 2 \cdot \pi \cdot f \cdot x)}{2} + \frac{e^{-\frac{x^2}{2} - \frac{y^2}{2}} \cdot \cos(2 \cdot \pi \cdot x + 2 \cdot \pi \cdot f \cdot x)}{2} \right) dx dy = \pi \cdot e^{-2 \cdot \pi^2 \cdot (f-1)^2} + \pi$$

```
plotfunc2d( vf, f=0..5 );
```



```
// Richtung
```

```
myplot( g );
```

```
hx:=(x,y)->exp(-x^2*3^2/2):
```

```
hy:=(x,y)->exp(-y^2*3^2/2):
```

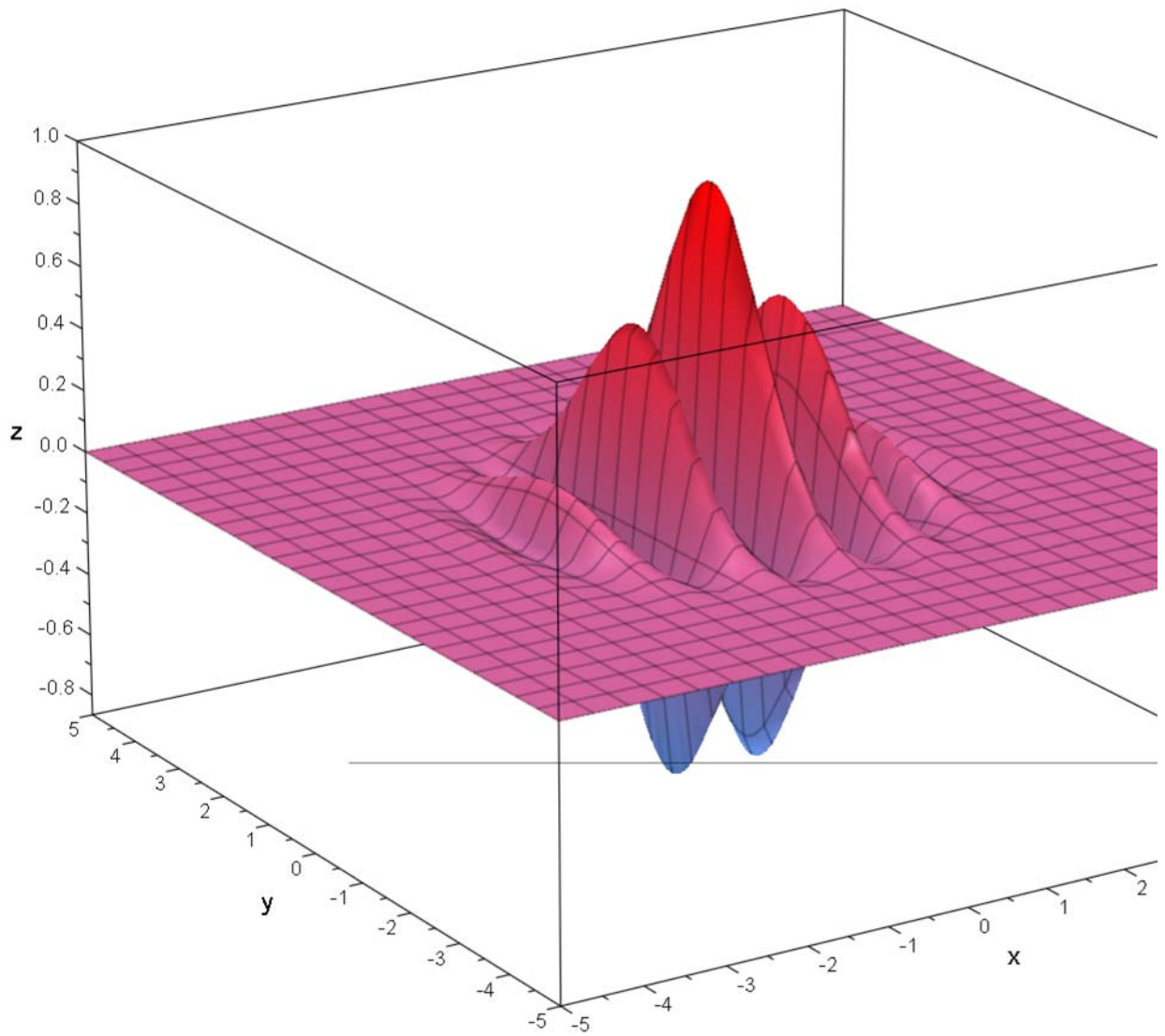
```
myplot( g*hx, hx/5 );
```

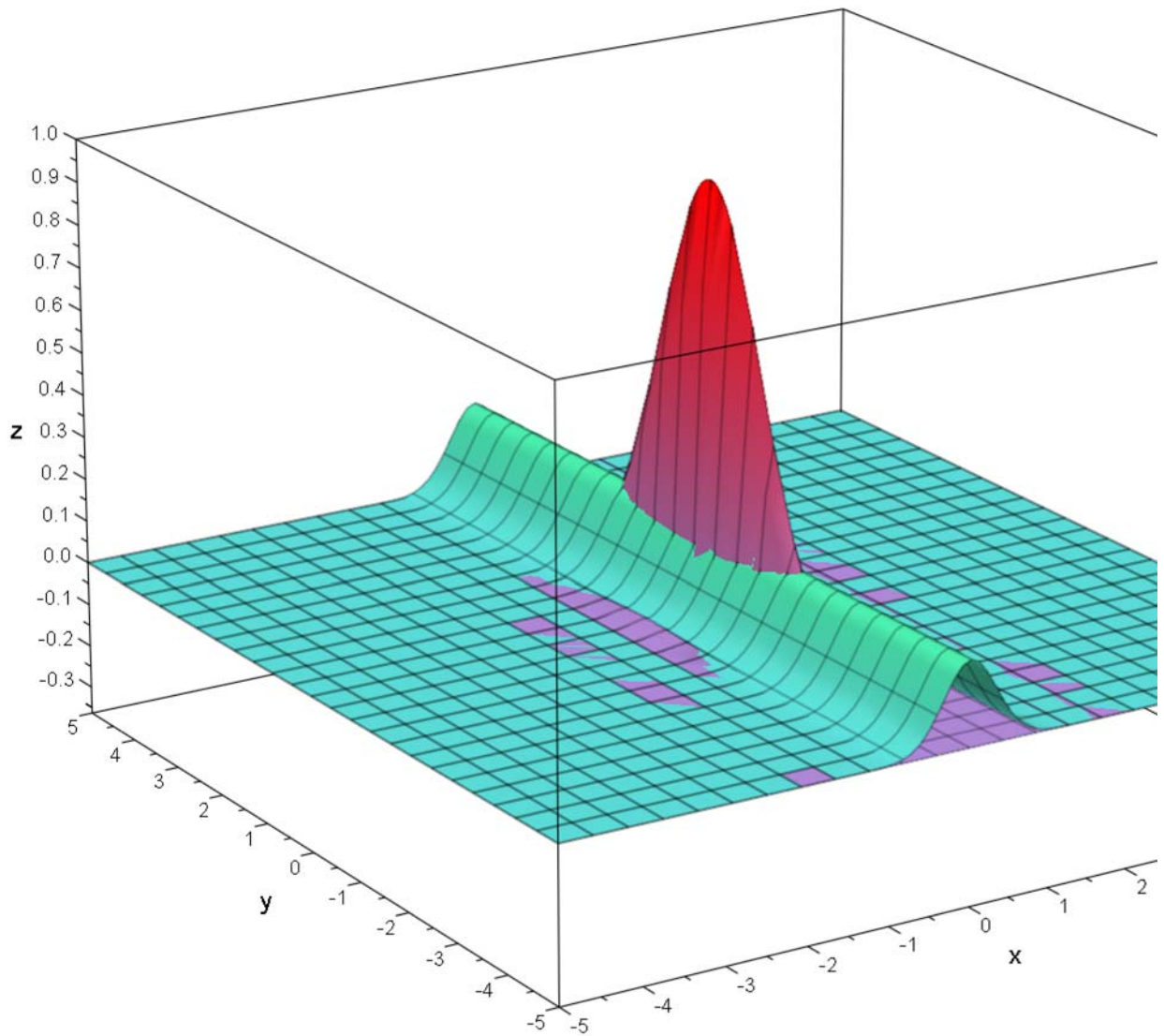
```
Simplify( int( int( Simplify( (g*hx)(x,y) ), x=-infinity..infinity), y=-infinity
```

```
myplot( g*hy, hy/5 );
```

```
Simplify( int( int( Simplify( (g*hy)(x,y) ), x=-infinity..infinity), y=-infinity
```

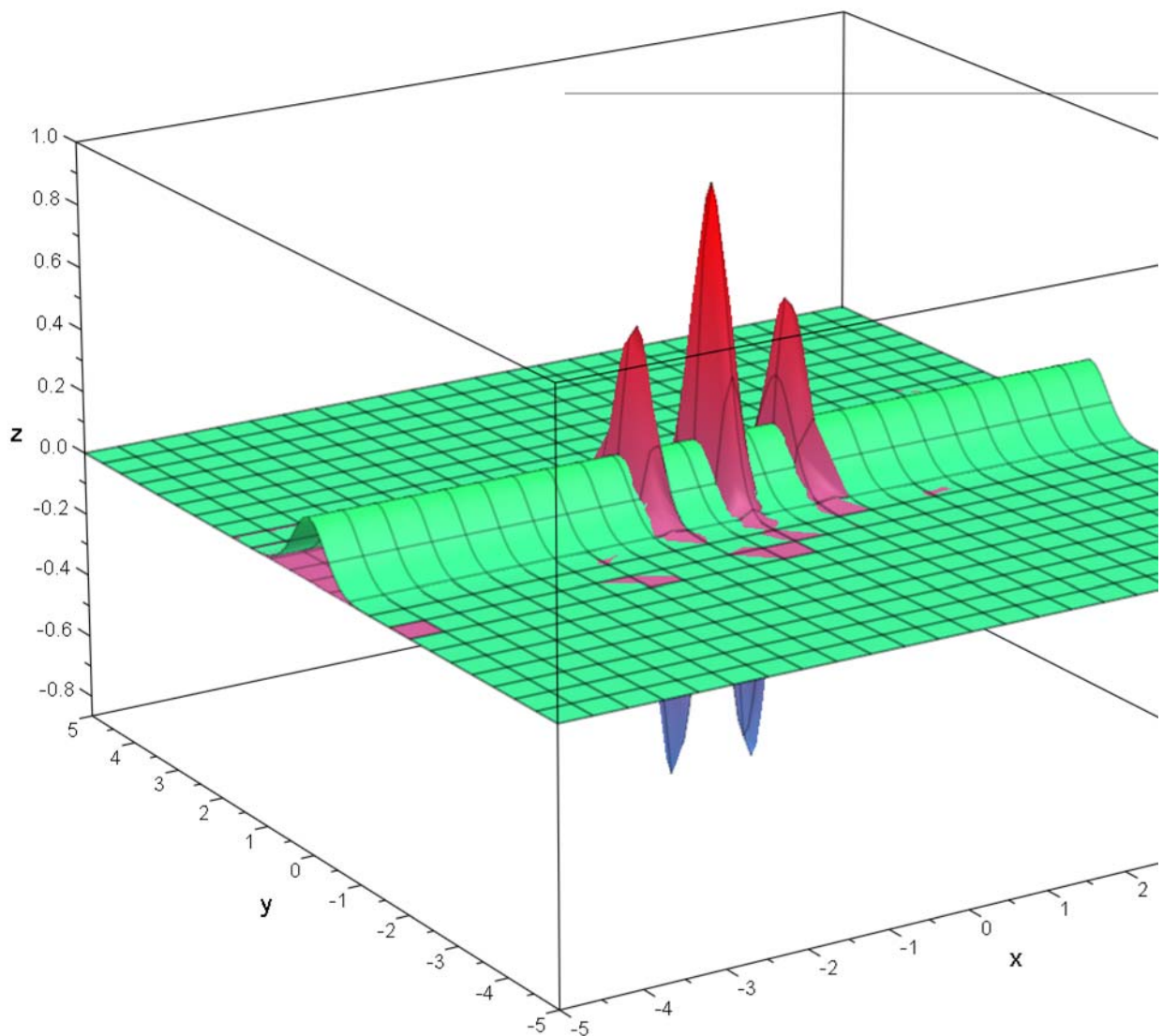
```
//myplot(3*g,hx,hy);
```





■ $((x, y) \rightarrow \cos(2 \cdot \pi \cdot x) \cdot \exp(-1/2 \cdot x^2 - 1/2 \cdot y^2)) \cdot (\frac{\pi \cdot \sqrt{10} \cdot e^{-\frac{\pi^2}{5}}}{5})$
■ $1/5 \cdot ((x, y) \rightarrow \exp(-(x^2 + 3 \cdot y^2)/2))$

$$\frac{\pi \cdot \sqrt{10} \cdot e^{-\frac{\pi^2}{5}}}{5} = 0.2760049827$$



■ $((x, y) \rightarrow \cos(2 \cdot \pi \cdot x) \cdot \exp(-1/2 \cdot x^2 - 1/2 \cdot y^2)) \cdot$
■ $1/5 \cdot ((x, y) \rightarrow \exp(-(y^2 \cdot 3^2)/2))$

$$\frac{\pi \cdot \sqrt{10} \cdot e^{-2 \cdot \pi^2}}{5} = 0.000000005315576937$$

```

sx:=gaborfilter(infinity,infinity,1);
hdelta:=fp::unapply( sx( (cos(2*PI*delta)*x+sin(2*PI*delta)*y,-sin(2*PI*delta)*x
(x, y) → cos(2 · π · x)
(delta, x, y) → cos(2 · π · (x · cos(2 · π · delta) + y · sin(2 · π · delta)))

```

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```

Simplify( simplify( g(x,y)*hdelta(d,x,y), sin ))
assuming d>0:

```



```
assuming d>0:
kd:=Simplify(combine(% ,exp)) assuming d>0;
```

$$e^{-\frac{x^2}{2}-\frac{y^2}{2}} \cdot \frac{(\cos(2 \cdot \pi \cdot (x \cdot \cos(2 \cdot \pi \cdot d) + y \cdot \sin(2 \cdot \pi \cdot d)) + 2 \cdot \pi \cdot x) + \cos(2 \cdot \pi \cdot (x \cdot \cos(2 \cdot \pi \cdot d) - x + y \cdot \sin(2 \cdot \pi \cdot d)))}{2}}$$

```
select([prog::find(kd,cos)],nops<=3):
map(% , z->collect( (op(kd,z[1..-2].[1])), [x,y], simplify) ) ;
```

$$[(2 \cdot \pi \cdot (\cos(2 \cdot \pi \cdot d) + 1)) \cdot x + (2 \cdot \pi \cdot \sin(2 \cdot \pi \cdot d)) \cdot y, (2 \cdot \pi \cdot (\cos(2 \cdot \pi \cdot d) - 1)) \cdot x + (2 \cdot \pi \cdot \sin(2 \cdot \pi \cdot d)) \cdot y]$$

```
prerotkd:=simplify(subs( kd, [x=cos(2*PI*delta)*x+sin(2*PI*delta)*y, y=-sin(2*PI
```

$$e^{-\frac{x^2}{2}-\frac{y^2}{2}} \cdot \frac{(\cos(2 \cdot \pi \cdot x \cdot \cos(2 \cdot \pi \cdot \delta) + 2 \cdot \pi \cdot y \cdot \sin(2 \cdot \pi \cdot \delta) - 2 \cdot \pi \cdot x \cdot \cos(2 \cdot \pi \cdot (d + \delta)) - 2 \cdot \pi \cdot y \cdot \sin(2 \cdot \pi \cdot (d + \delta)))}{2}}$$

```
select([prog::find(prerotkd,cos)],nops<=3):
map(% , z->collect( (op(prerotkd,z[1..-2].[1])), [x,y], simplify) ) ;
```

$$[(-2 \cdot \pi \cdot (\cos(2 \cdot \pi \cdot (d + \delta)) - \cos(2 \cdot \pi \cdot \delta))) \cdot x + (2 \cdot \pi \cdot (\sin(2 \cdot \pi \cdot \delta) - \sin(2 \cdot \pi \cdot (d + \delta)))) \cdot y, (2 \cdot \pi \cdot (\cos(2 \cdot \pi \cdot (d + \delta)) + \cos(2 \cdot \pi \cdot \delta))) \cdot x + (2 \cdot \pi \cdot (\sin(2 \cdot \pi \cdot \delta) + \sin(2 \cdot \pi \cdot (d + \delta)))) \cdot y]$$

```
rotkd:=simplify(subs(prerotkd,delta=-d/2));
```

$$e^{-\frac{x^2}{2}-\frac{y^2}{2}} \cdot \frac{(\cos(4 \cdot \pi \cdot x \cdot \cos(\pi \cdot d)) + \cos(4 \cdot \pi \cdot y \cdot \sin(\pi \cdot d)))}{2}}$$

```
rotkd2:=1/2*(gaborfilter(1,1,2*cos(PI*d))(x,y)+gaborfilter(1,1,2*sin(PI*d))(y,x)
simplify(rotkd-rotkd2)
```

$$\frac{\cos(4 \cdot \pi \cdot x \cdot \cos(\pi \cdot d)) \cdot e^{-\frac{x^2}{2}-\frac{y^2}{2}}}{2} + \frac{\cos(4 \cdot \pi \cdot y \cdot \sin(\pi \cdot d)) \cdot e^{-\frac{x^2}{2}-\frac{y^2}{2}}}{2}$$

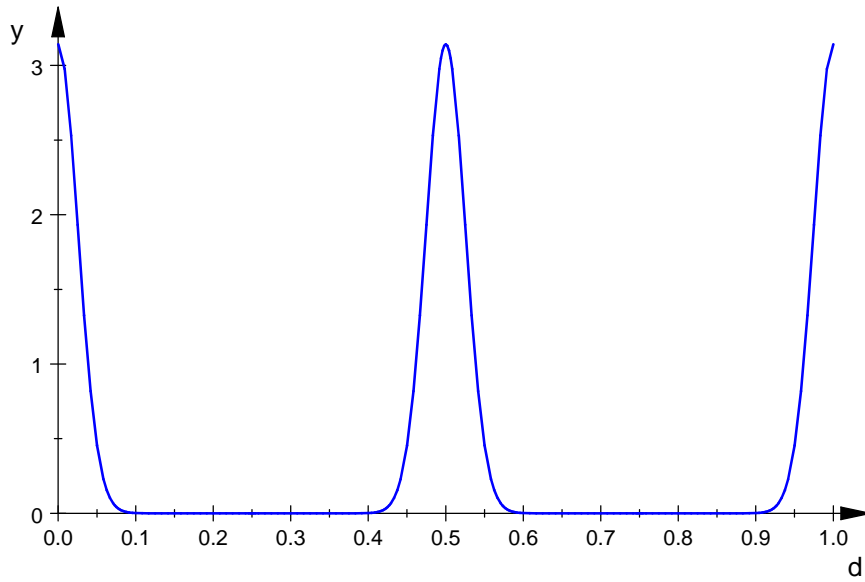
0

```
hold(int)( hold(int)( kd, x=-infinity..infinity), y=-infinity..infinity)
= (vd:=1/2*(Igaborfilter(1,1,2*cos(PI*d))+Igaborfilter(1,1,2*sin(PI*d))));
```

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}-\frac{y^2}{2}} \cdot \frac{(\cos(2 \cdot \pi \cdot (x \cdot \cos(2 \cdot \pi \cdot d) + y \cdot \sin(2 \cdot \pi \cdot d)) + 2 \cdot \pi \cdot x) + \cos(2 \cdot \pi \cdot (x \cdot \cos(2 \cdot \pi \cdot d) - x + y \cdot \sin(2 \cdot \pi \cdot d)))}{2}} dx dy$$

```
plotfunc2d( vd, d=0..1 );
```





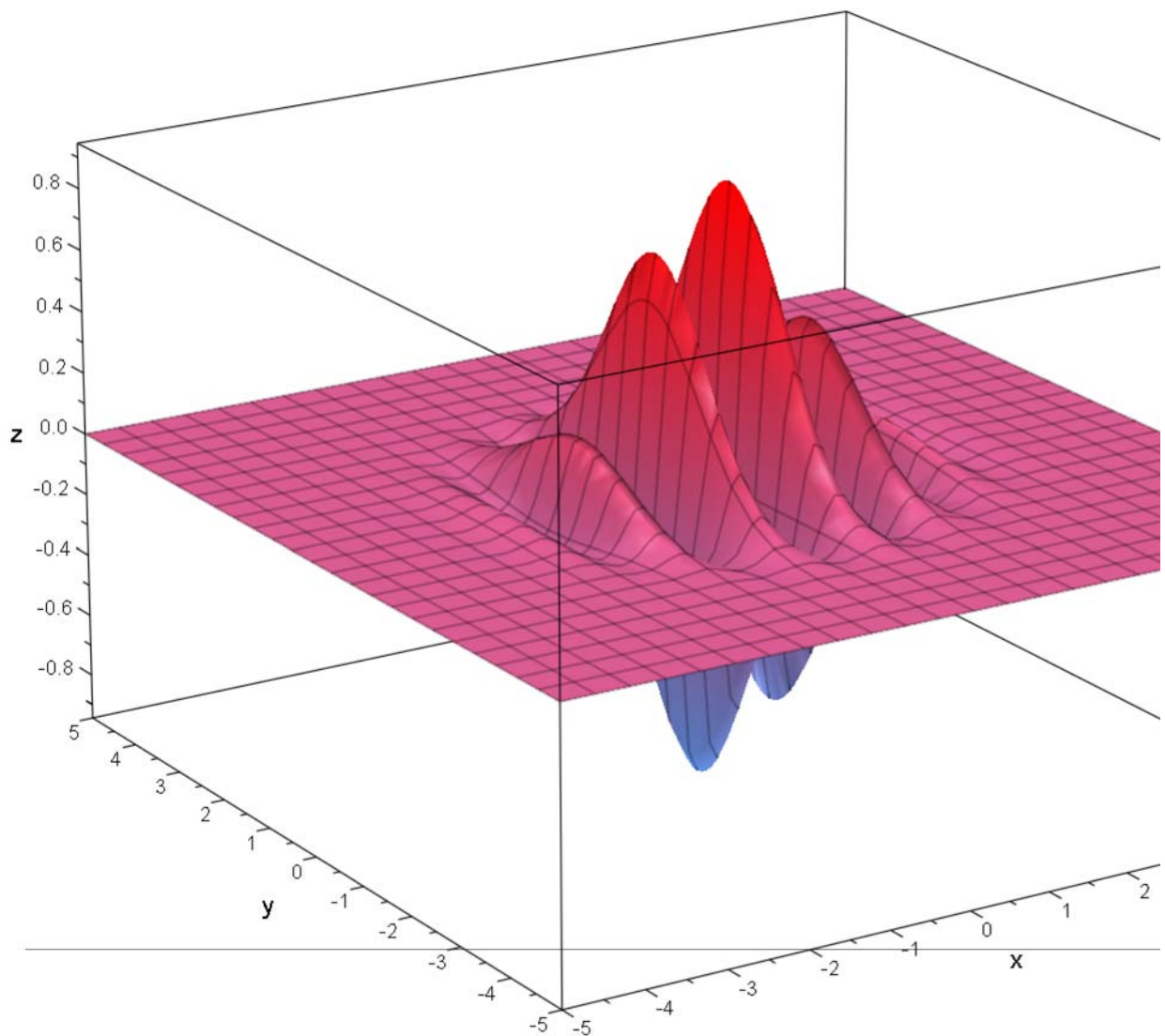
```

ksin:=subs( gaborfilter(a,b,f)(x,y), cos(2*PI*f*x)=sin(2*PI*f*x) );
myplot( subs(ksin,a=1,b=1,f=1) );
Simplify( int(
  Simplify( int(
    ksin,
    y=-infinity..infinity ) ),
  x=-infinity..infinity ) )

```

assuming a>0 assuming b>0 assuming

$$\sin(2 \cdot \pi \cdot f \cdot x) \cdot e^{-\frac{x^2}{2 \cdot a^2} - \frac{y^2}{2 \cdot b^2}}$$



0

// **Verschiebung**

```

vs:=gaborfilter(infinity,infinity,1)(x+s,y);
ks0:=simplify(simplify(vs*g(x,y),sin));
eval( subs( hold(simplify)(cos(a+b))=expand(cos(a+b)), a=2*PI*s, b=2*PI*2*x ) );
ks:=subs( ks0, % );
ks2:=gaborfilter(1,1,0)(x,y)*cos(2*PI*s)/2+gaborfilter(1,1,2)(x,y)*cos(2*PI*s)*1/2;
simplify(ks-ks2);
hold(int)(hold(int)( ks, y=-infinity..infinity), x=-infinity..infinity )
= (vs:=Igaborfilter(1,1,0)*cos(2*PI*s)/2+Igaborfilter(1,1,2)*cos(2*PI*s)*1/2);
cos(2·π·(s+x))

```

$$e^{-\frac{x^2}{2}-\frac{y^2}{2}} \cdot (\cos(2 \cdot \pi \cdot s) + \cos(2 \cdot \pi \cdot (s + 2 \cdot x)))$$

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$$e^{-\frac{x^2}{2}-\frac{y^2}{2}} \cdot (\cos(2 \cdot \pi \cdot s) + \cos(2 \cdot \pi \cdot (s + 2 \cdot x)))$$

$$\cos(2 \cdot \pi \cdot (s + 2 \cdot x)) = \cos(2 \cdot \pi \cdot s) \cdot \cos(4 \cdot \pi \cdot x) - \sin(2 \cdot \pi \cdot s) \cdot \sin(4 \cdot \pi \cdot x)$$

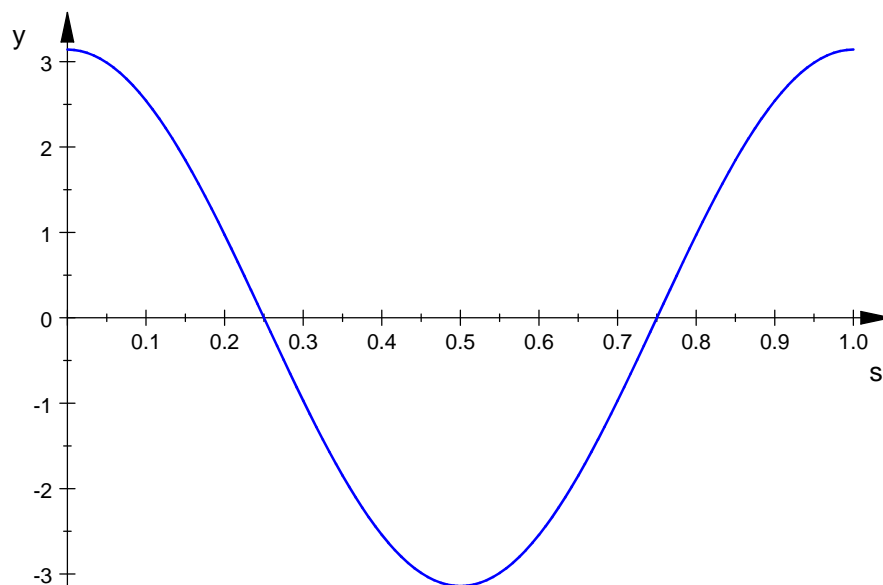
$$e^{-\frac{x^2}{2}-\frac{y^2}{2}} \cdot (\cos(2 \cdot \pi \cdot s) - \sin(2 \cdot \pi \cdot s) \cdot \sin(4 \cdot \pi \cdot x) + \cos(2 \cdot \pi \cdot s) \cdot \cos(4 \cdot \pi \cdot x))$$

$$\frac{\cos(2 \cdot \pi \cdot s) \cdot e^{-\frac{x^2}{2}-\frac{y^2}{2}}}{2} + \frac{\cos(2 \cdot \pi \cdot s) \cdot \cos(4 \cdot \pi \cdot x) \cdot e^{-\frac{x^2}{2}-\frac{y^2}{2}}}{2} - \frac{\sin(2 \cdot \pi \cdot s) \cdot \sin(4 \cdot \pi \cdot x) \cdot e^{-\frac{x^2}{2}-\frac{y^2}{2}}}{2}$$

0

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-\frac{x^2}{2}-\frac{y^2}{2}} \cdot (\cos(2 \cdot \pi \cdot s) - \sin(2 \cdot \pi \cdot s) \cdot \sin(4 \cdot \pi \cdot x) + \cos(2 \cdot \pi \cdot s) \cdot \cos(4 \cdot \pi \cdot x))}{2} dy dx = \pi \cdot \cos(2 \cdot \pi \cdot s) + \pi$$

```
plotfunc2d( vs, s=0..1 );
```



```
t:=(time()-t0)*0.001*unit::s;
```

```
531.835 · s
```

```
unit::convert(t,unit::min)
```

```
8.863916667 · min
```

[