Electronic passport and biometrics, winter 2006

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10. Exercise sheet
Hand in solutions until Tuesday, 30 January, $12^{\underline{15}}$.

Exercise 10.1 (Experiment). (6+1 points)

Given a function h it is claimed that the formula

$$f(h) = \frac{V(h)}{2\pi\alpha(h)}$$

with

$$V(h) = \frac{1}{2k}\int_{-k}^{k} |h'(x)| \, \mathrm{d}x, \text{ (variation)}$$

$$\alpha(h) = \frac{1}{2k}\int_{-k}^{k} |h(x) - E(h)| \, \mathrm{d}x \text{ (average amplitude), and}$$

$$E(h) = \frac{1}{2k}\int_{-k}^{k} h(x) \, \mathrm{d}x \text{ (mean value)}$$

gives the desired average frequency.

(i) Use functions $h_f(x) = \cos(2\pi f \cdot x)$. Calculate (numerically) its average frequency $f(h_f)$. Use $k = 1$ with $f \in_R [0, 10]$ and compare $f(h)$ with f. (You might have problems if $k \cdot f \geq 4\ldots$) 4

(ii) Take again the wave $h(x) = \cos(2\pi f \cdot x)$. Derive the formula:

- Now express $h'(x)$ as a multiple of a shift $h(x + \delta)$ of h, so $h'(x) = \varphi \cdot h(x + \delta)$. Integrate absolute value of the formula over an interval $-k..k$. 2
- Assuming that $k \gg \delta$, we can shift the integration interval on the right by δ without losing much. +1

Exercise 10.2 (Poincaré index). (6 points)

Choose random values for the direction in the cells around a given point. (Choose eight angles!) The direction here indicates the direction of an *oriented* ridge through that point. In some cases mark one (say, the first one) of them as 'unorientable', ie. we assume the value to be known only up to 180°. Compute the Poincaré index $P = \sum_{0 \leq k < 8} \mathrm{angle}(d_k, d_{k+1})$. Clearly, $\mathrm{angle}(\varphi, \psi)$ is related to $\varphi - \psi$ but is always between $-180°$ and $180°$.

(i) Repeat the experiment some times and 3

(ii) discuss your results. (You might want to draw some pictures to interpret them.) 3

(Actually, this is somehow a discretization of $\frac{1}{2\pi i} \oint_\gamma \frac{f'(z)}{f(z)} \, \mathrm{d}z$ which counts how often a continuous path γ loops around zeroes of a complex function f.)