Assignment 3

1 Mathematical properties of binary trees

Property 1. *The internal path length of a binary tree with N internal nodes is at least $N \log(\frac{N}{4})$ and at most $N(N-1)/2$*

Proof. The worst case and best case are achieved for the same trees referred to in the discussion of a binary tree's height's bounds, namely, the degenerate tree and the balanced tree.

The internal path length of the worst case tree is $0 + 1 + \cdots + N - 1 = N(N-1)/2$.

The best case tree has $N + 1$ external nodes at height no more that $\log N$. Multiplying these and applying the property that relates the external path of tree with its internal path we get the bound $(N+1)\log N - 2N < N log(\frac{N}{4})$

□

2 Tree traversal

- **Preorder: node left right**
  $n_0 n_1 n_3 n_6 n_7 n_4 n_8 n_{10} n_{12} n_2 n_5 n_9 n_{11} n_{13}$

- **Inorder: left node right**
  $n_6 n_3 n_7 n_1 n_{12} n_{10} n_8 n_4 n_0 n_5 n_9 n_{13} n_{11} n_2$

- **Postorder: left right node**
  $n_6 n_7 n_3 n_{12} n_{10} n_8 n_4 n_1 n_{13} n_{11} n_9 n_5 n_2 n_0$