B-it **Bridging Course (SS05)**

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Assignment 2

1 Recursive lists

1. **Concatenation of lists**
   We define concat: list $\times$ list $\rightarrow$ list as the concatenation of two lists.

   - length(concat(l, m)) = length(l) + length(m)
   - i^{th}(concat(l, m), j) = i^{th}(l,j) if $1 \leq$ j $\leq$ length(l), i^{th}(m,j-length(l)) otherwise.
   - concat(empty_list, l) = l
   - concat(cons(e, l), m) = cons(e,concat(l,m))

2. **Search of an element (present) in a list**
   We define search: list $\times$ element $\rightarrow$ position. Complete the following axioms:

   - content(search(l, e))= e
   - search(cons(e,l), e) = head(cons(e,l))
   - e $\neq$ f, search(cons(e, l), f)= succ(search(l,f))

2 Mathematical properties of binary trees

Property 1. *A binary tree with N internal nodes has 2N links: N-1 links to internal nodes and N+1 links to external nodes.*

Proof. In any tree, each node except the root, has a unique parent, and every edge connects a node to its parent, so there are N-1 links connecting internal nodes. Similarly, each of the N +1 external nodes has one link, to its unique parent. □