7. Kryptotag – November 9th, 2007

Arithmetic operators for pairing-based cryptography

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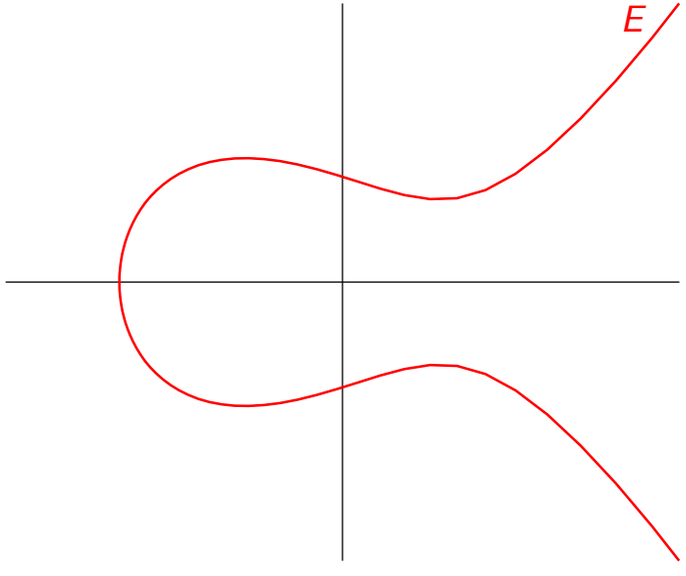
Outline of the talk

- Pairings?
- Arithmetic over $\mathbb{F}_{3^m}$
- Unified operator
- Results
- Final thoughts

Elliptic curves

- E defined by an equation of the form
  $y^2 = x^3 + Ax + B$



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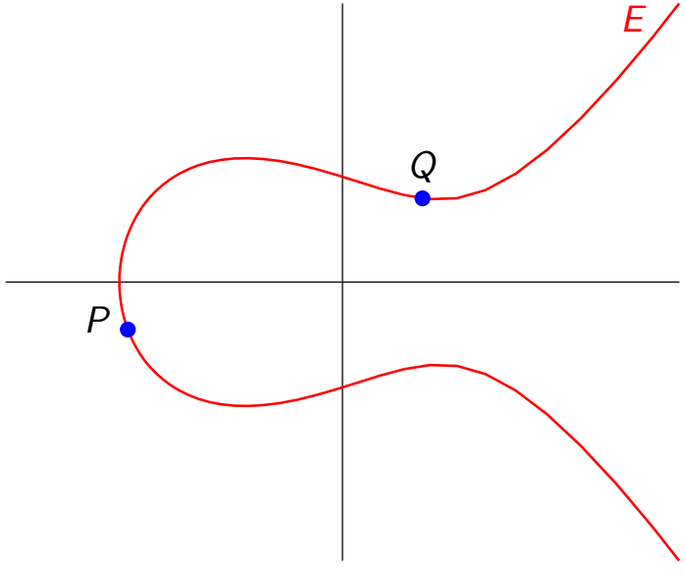

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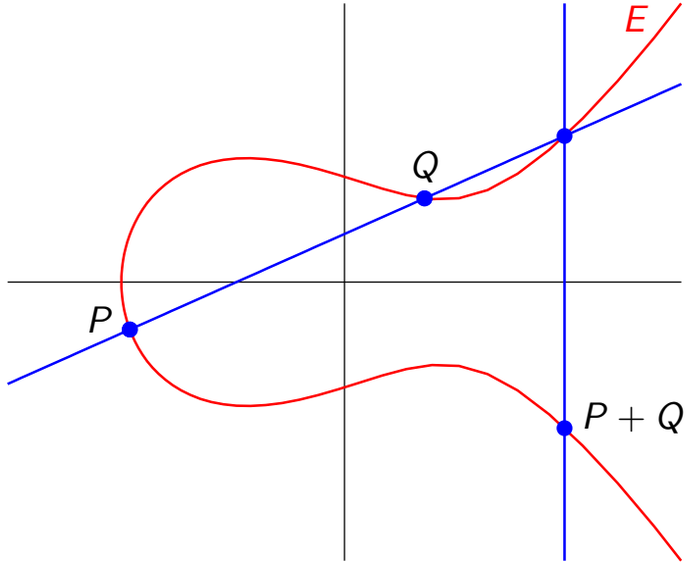

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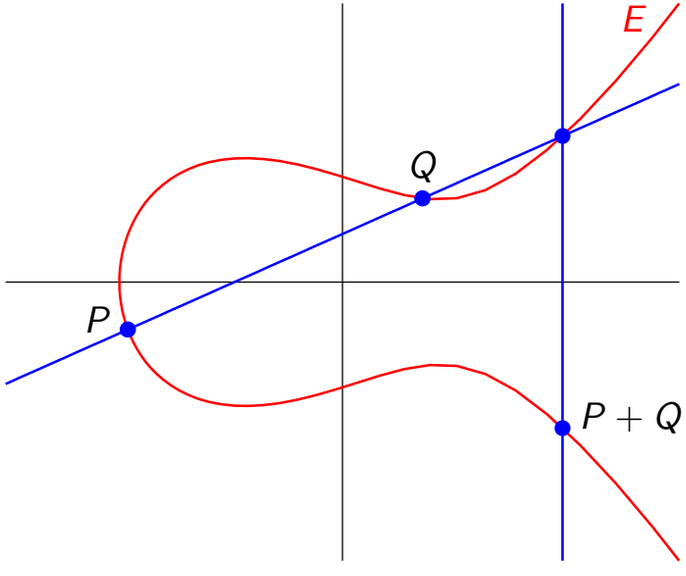

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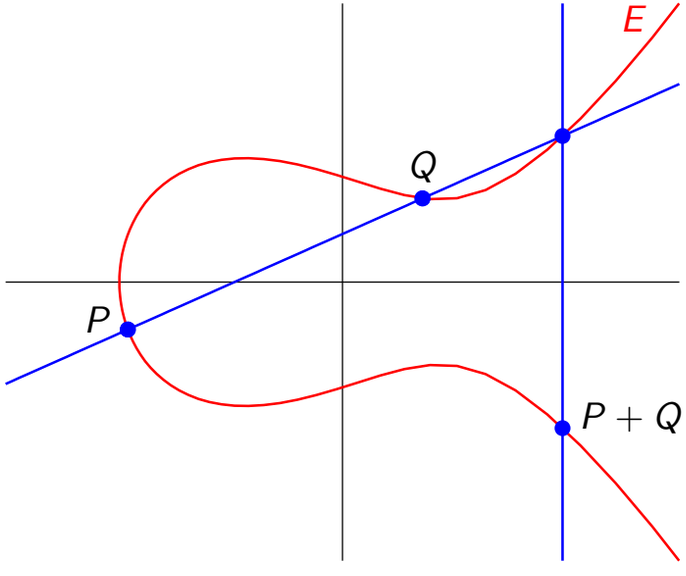
- But there's more: bilinear pairings



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that satisfies the following conditions:

- computability: $\hat{e}$ can be efficiently computed
- non-degeneracy: $\hat{e}(P, P) \neq 1_{G_2}$
- bilinearity: for all Q_1, Q_2 and $R \in G_1$,

$$\hat{e}(Q_1 + Q_2, R) = \hat{e}(Q_1, R)\hat{e}(Q_2, R) \quad \text{and} \quad \hat{e}(R, Q_1 + Q_2) = \hat{e}(R, Q_1)\hat{e}(R, Q_2)$$

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- Immediate property: for any integer a,

$$\hat{e}(aP, P) = \hat{e}(P, aP) = \hat{e}(P, P)^a$$

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- One-round three-party key agreement (Joux, 2000)
- Identity-based encryption
  - Boneh-Franklin, 2001
  - Sakai-Kasahara, 2001
  - ...
- Short signatures
  - Boneh-Lynn-Shacham (BLS), 2001
  - Zang-Safavi-Naini-Susilo (ZSS), 2004
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Operation	Count	$m = 97$
$+/-$	$121\lfloor\frac{m}{4}\rfloor + 186$	3090
$\times$	$25\lfloor\frac{m}{4}\rfloor + 79$	679
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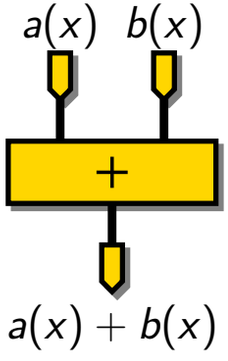
- Arithmetic over $\mathbb{F}_{3^m}$:
  - Polynomial basis: $\mathbb{F}_{3^m} \cong \mathbb{F}_3[x]/(f(x))$
  - Degree-m irreducible polynomial $f(x)$ carefully chosen

Addition over $\mathbb{F}_{3^m}$



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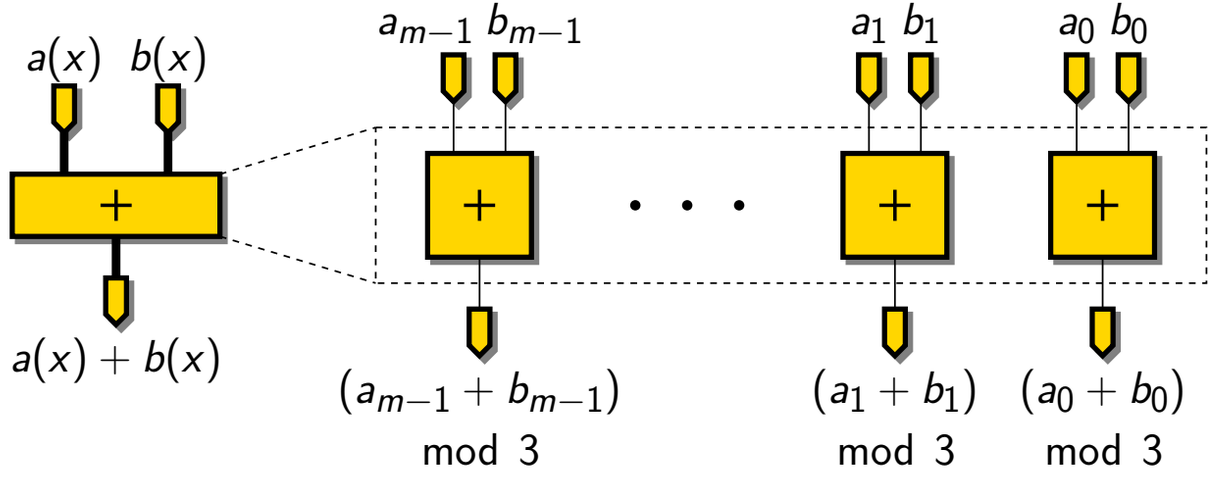

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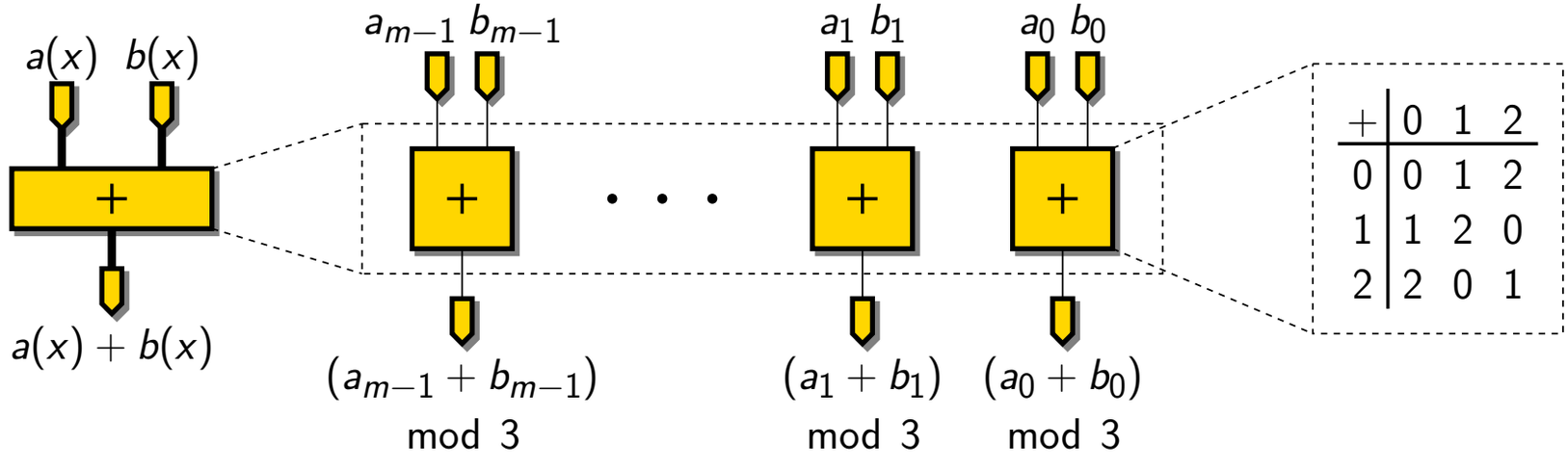
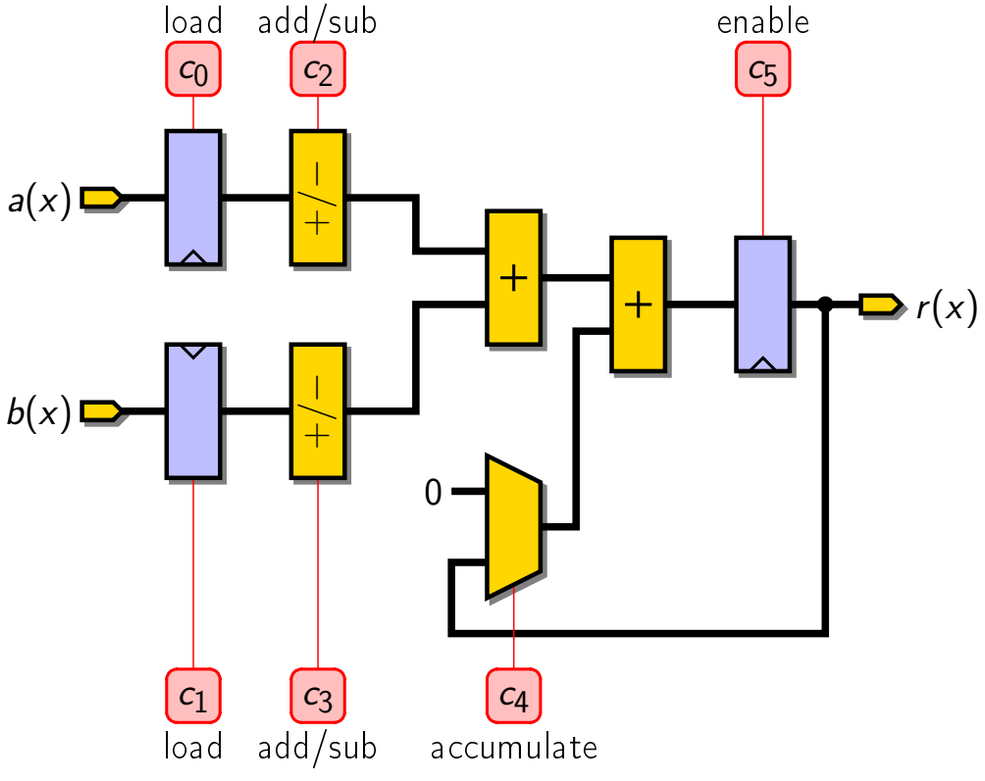


- coefficient-wise addition over $\mathbb{F}_3$
- addition over $\mathbb{F}_3$: small look-up table

Addition, subtraction and accumulation over $\mathbb{F}_{3^m}$



- sign selection: multiplication by 1 or 2

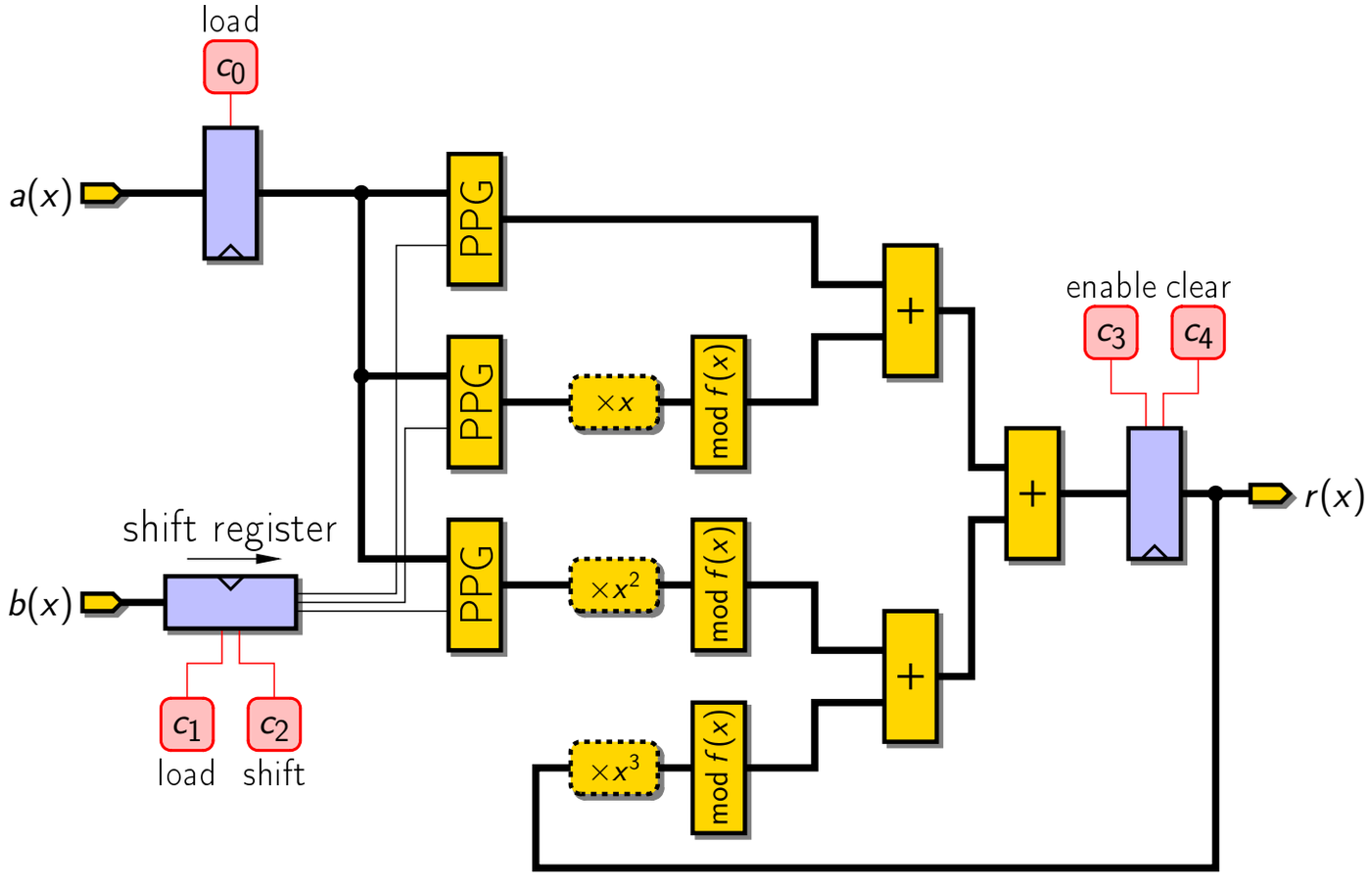
$$-a(x) \equiv 2a(x) \pmod 3$$

- feedback loop for accumulation

Multiplication over $\mathbb{F}_{3^m}$

- Parallel-serial multiplication
  - multiplicand loaded in a parallel register
  - multiplier loaded in a shift register
- Most significant coefficients first
- D coefficients processed at each iteration: $\left\lceil \frac{m}{D} \right\rceil$ iterations per multiplication

Multiplication over $\mathbb{F}_{3^m}$



- partial product generator (PPG): m multiplications over $\mathbb{F}_3$
- multiplication by x^i: only wiring
- simple modular reductions

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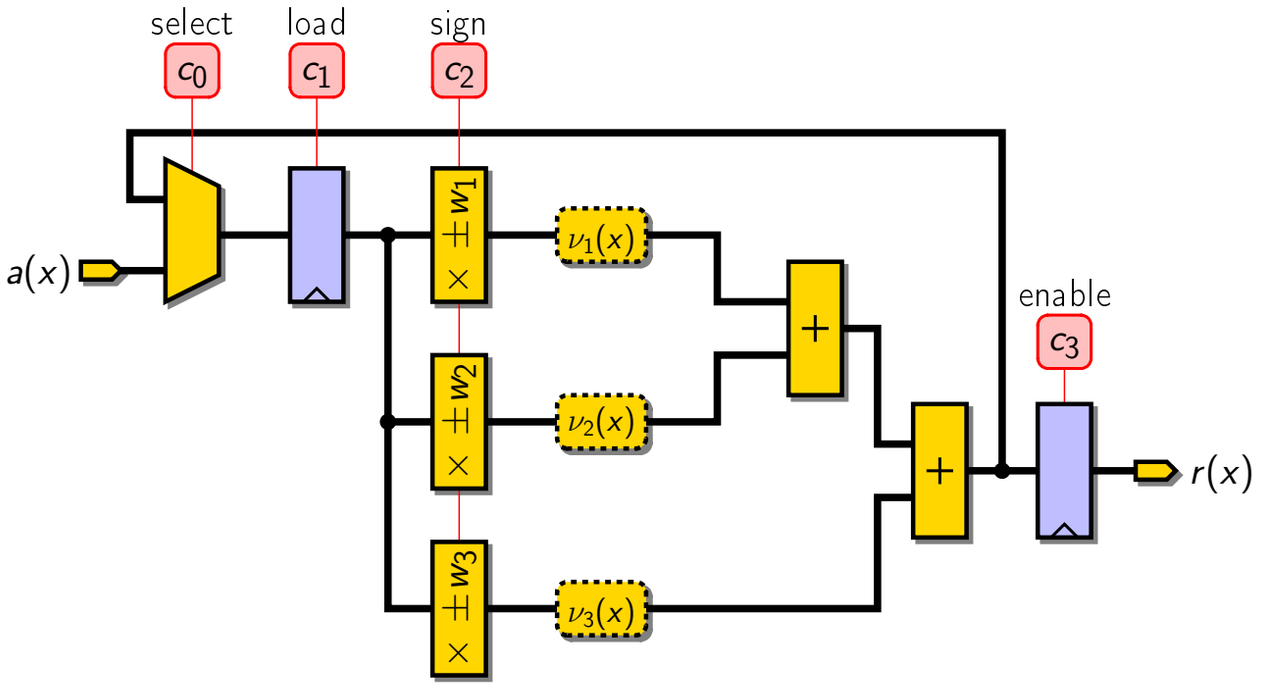
= \quad & w_1 \cdot \nu_1(x) + w_2 \cdot \nu_2(x) + w_3 \cdot \nu_3(x)
\end{aligned}
$$

- Required hardware:
  - only wires to compute the $\nu_i(x)$'s
  - possibly multiplications over $\mathbb{F}_3$
  - multi-operand addition over $\mathbb{F}_{3^m}$

Cubing over $\mathbb{F}_{3^m}$



- feedback loop for successive cubings
- sign selection for computing either $a(x)^3$ or $-a(x)^3$

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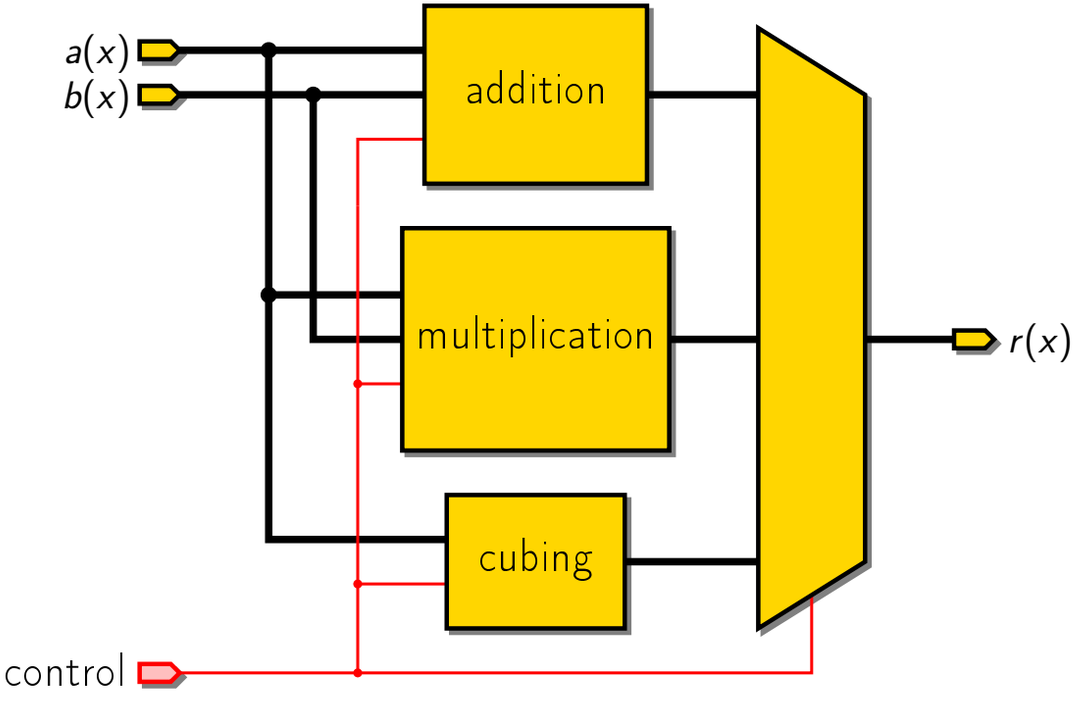
  - only one inversion for the full pairing: delay overhead is negligible ($< 1\%$)

The full processing element



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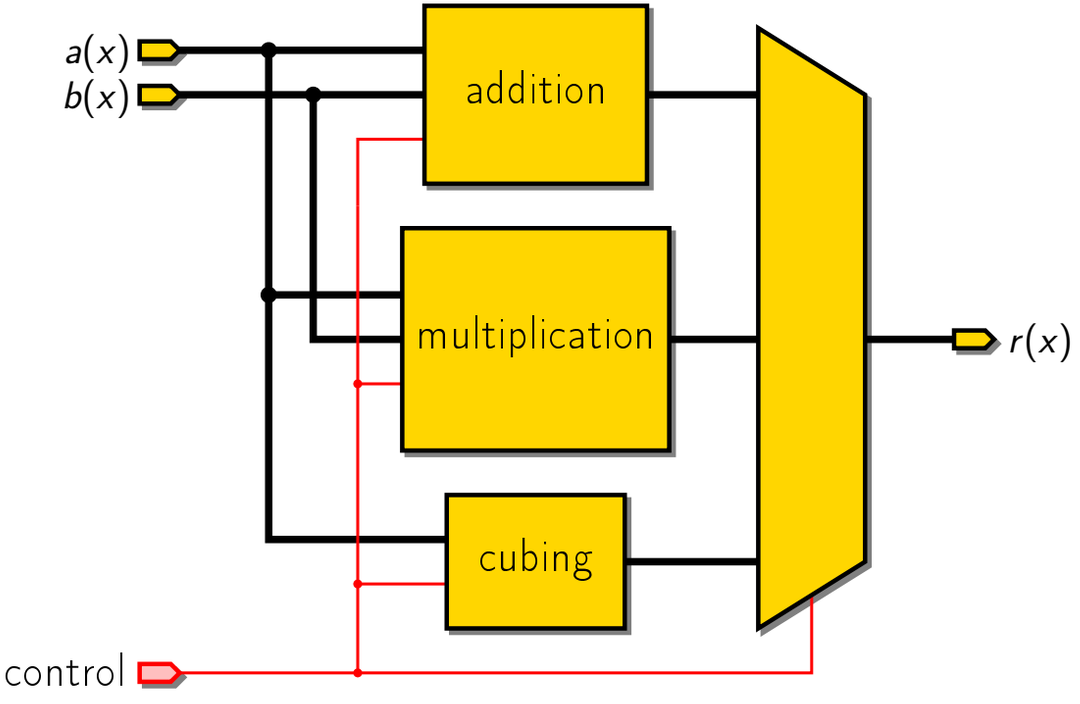
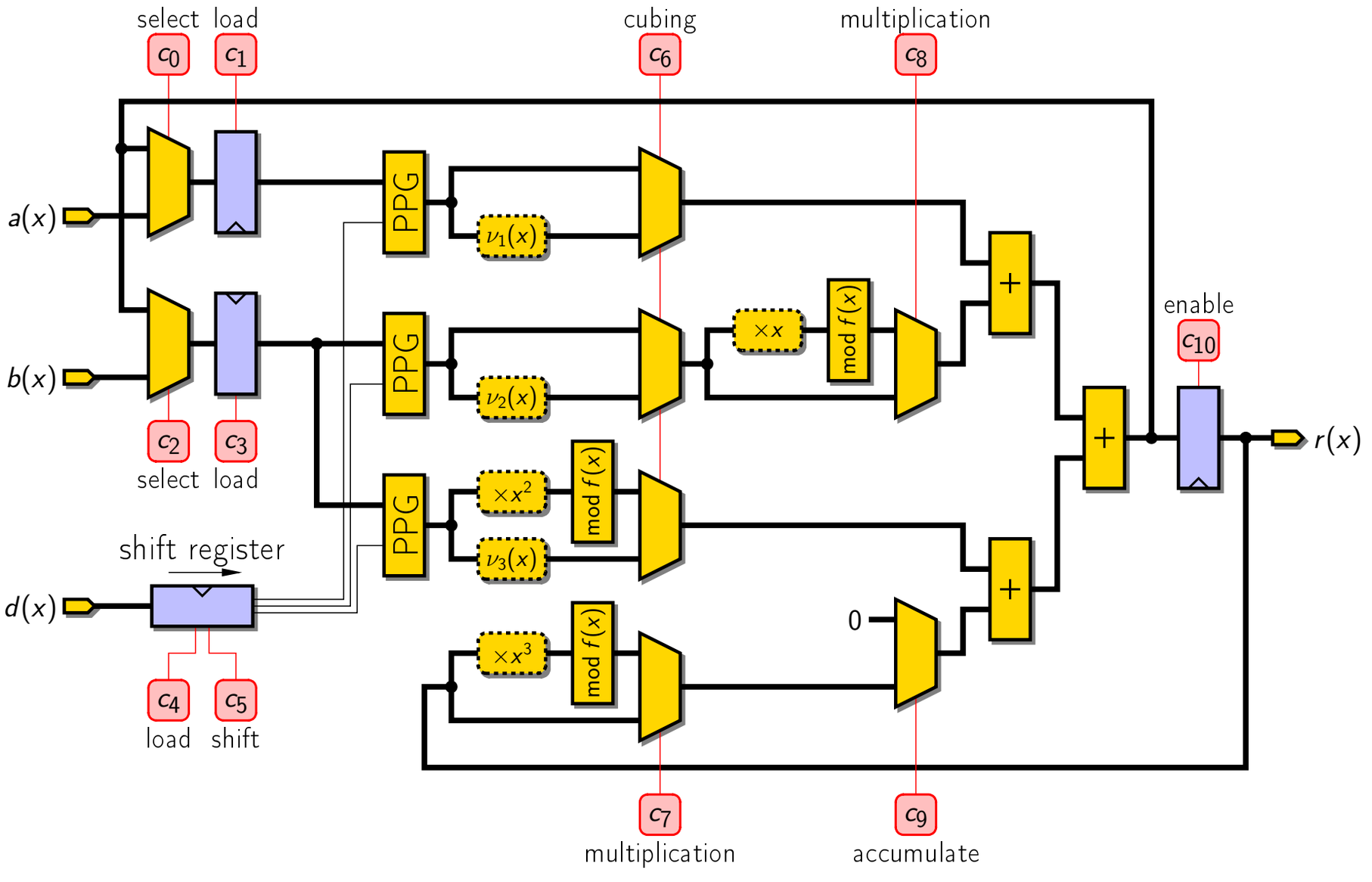


- For the η_T pairing:
  almost no parallelism between additions, multiplications and cubings
- Can we share hardware resources between the three operators?

Unified operator



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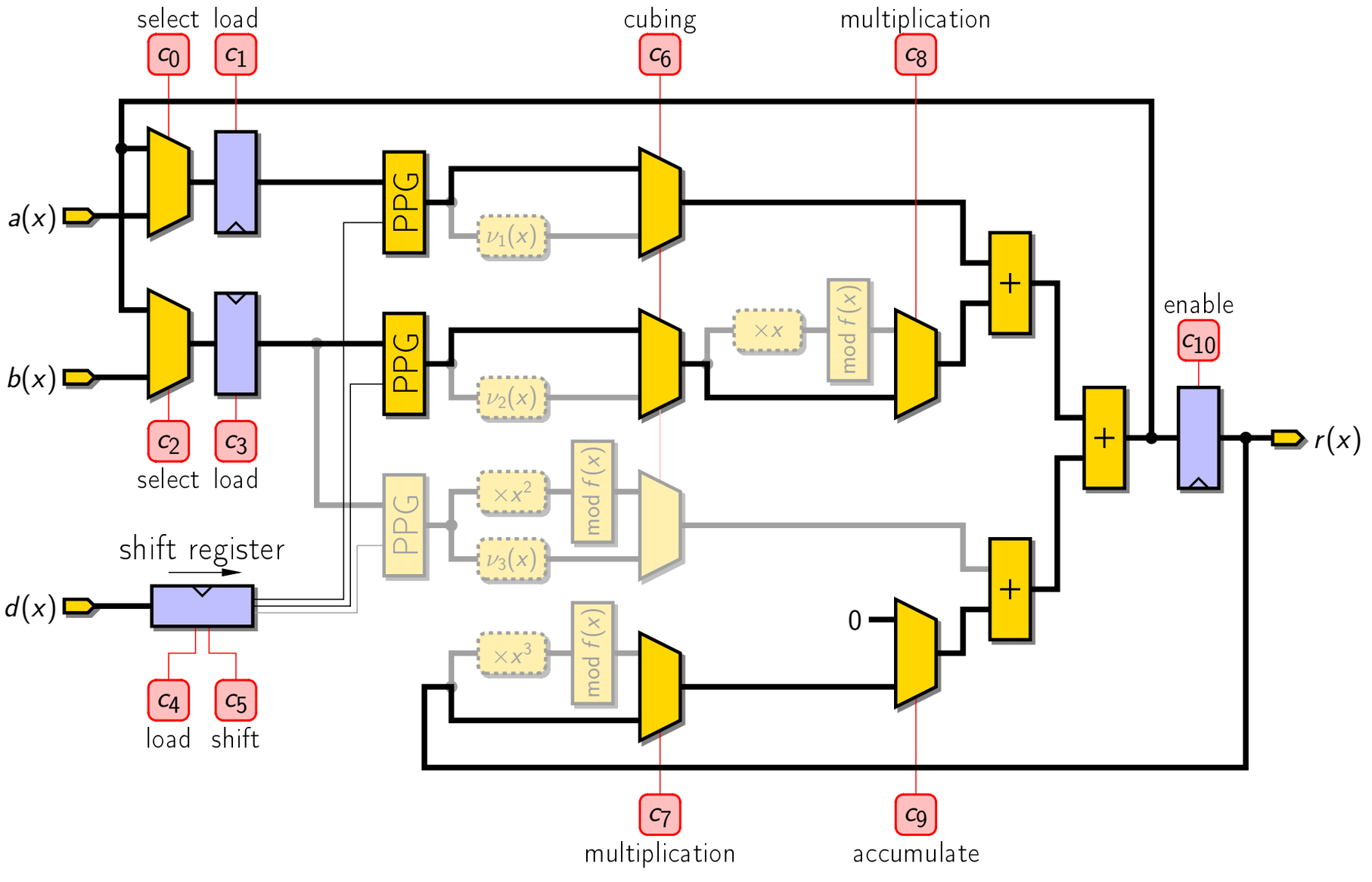

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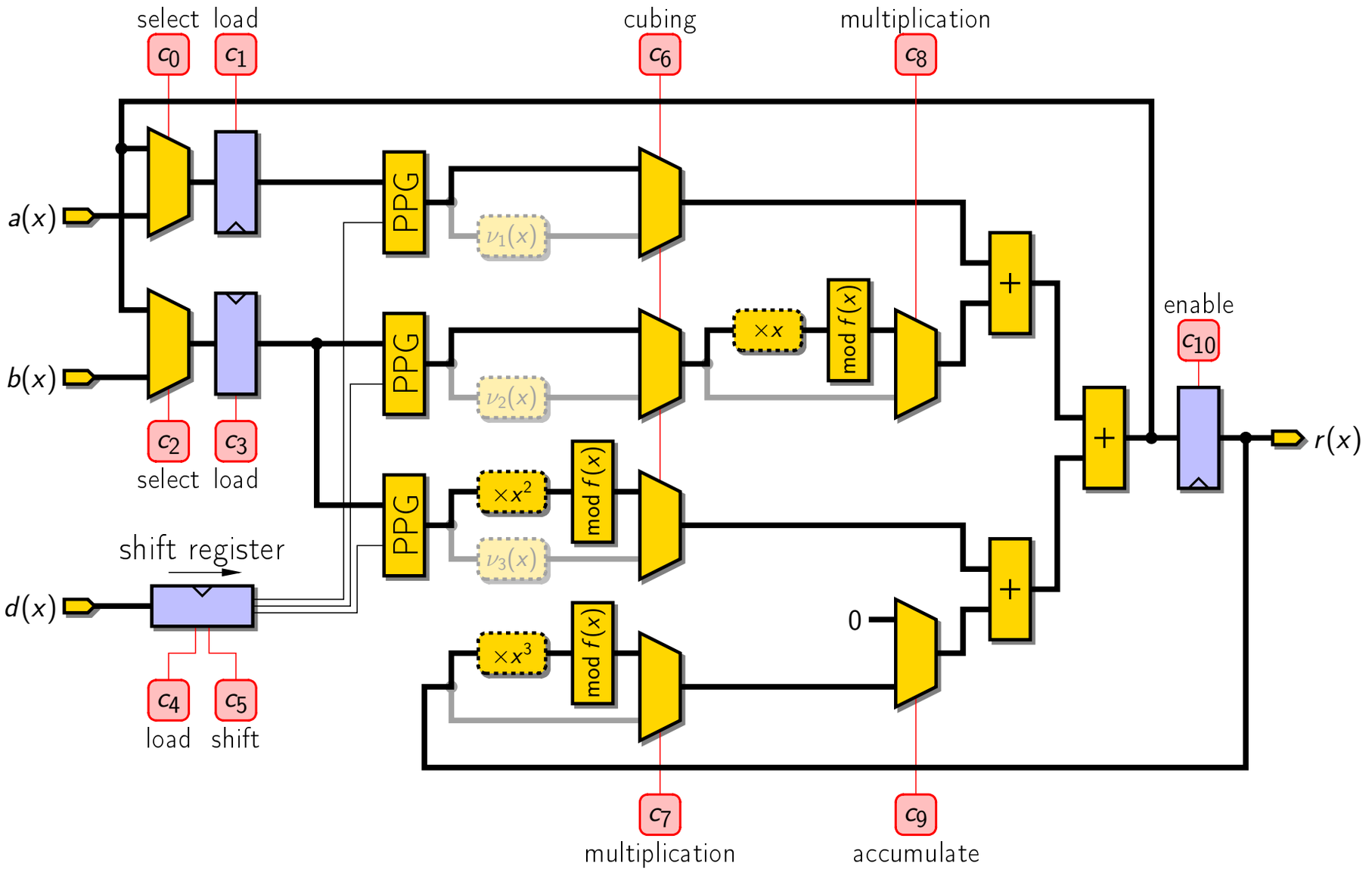

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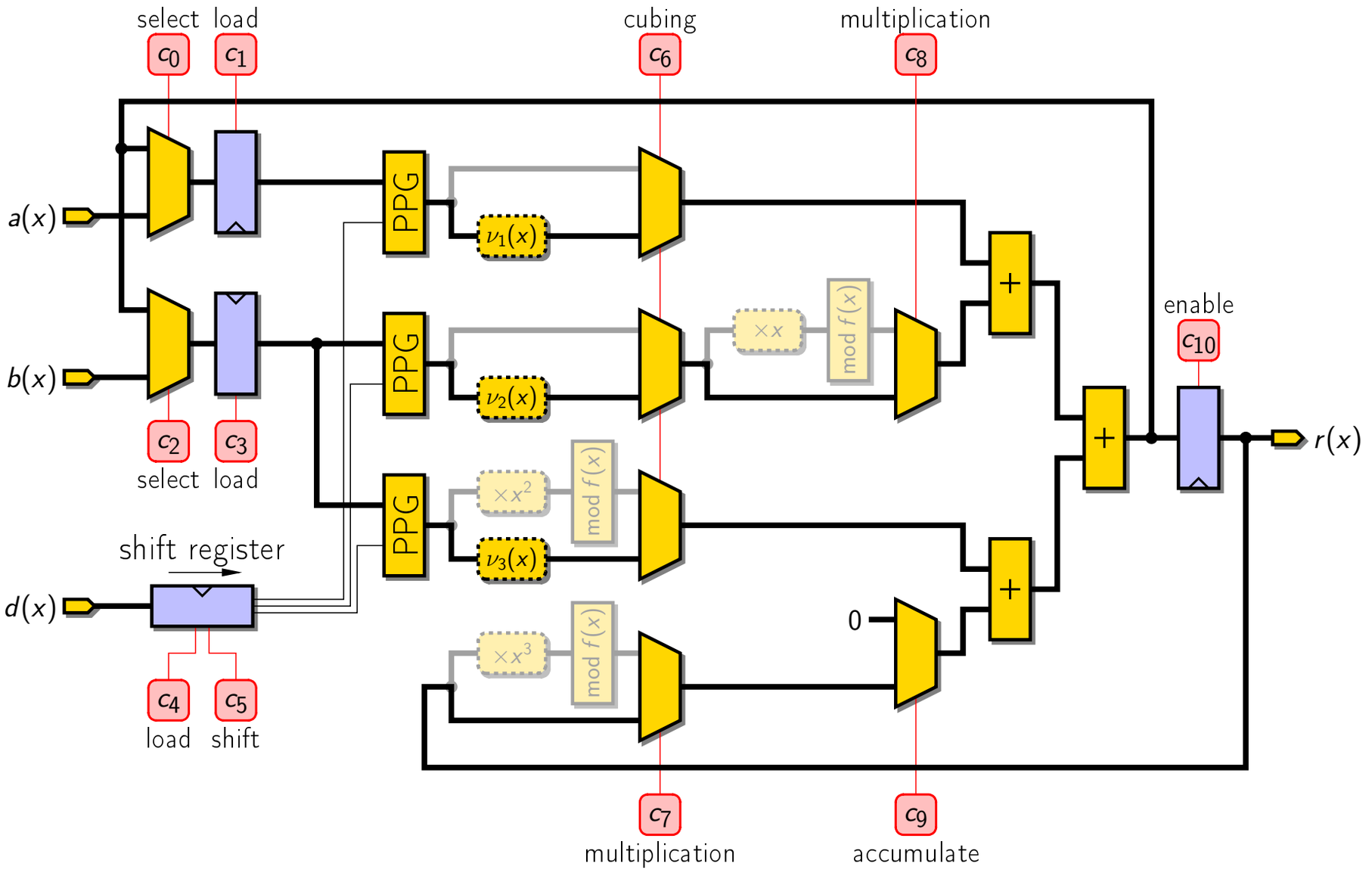

Results

- Full co-processor for computation of the η_T pairing
  - field: $\mathbb{F}_{3^{97}}$ with $f(x) = x^{97} + x^{12} + 2$
  - processing element: unified operator with $D = 3$
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- Area: 1833 slices (63% of the FPGA) and 6 memory blocks (21%)
- Clock frequency: 145 MHz
- Full η_T pairing: 27800 clock cycles
- Computation time: 192 μs

Comparisons

Architecture	Area	Time	FPGA
Proposed solution (CHES'07)	1833 slices	192 μs	Virtex-II Pro
Grabher and Page (CHES'05)	4481 slices	432 μs	Virtex-II Pro
Kerins *et al.* (CHES'05)	55616 slices	850 μs	Virtex-II Pro
Ronan *et al.* (ITNG'07)	10000 slices	178 μs	Virtex-II Pro
Beuchat *et al.* (Arith'07 & WAIFI'07)	18000 LEs ($\approx$ 9000 slices)	33 μs	Cyclone II
Jiang (Univ. Honk Kong, 2007)	74105 slices	21 μs	Virtex-4 LX

Conclusion

- Unified operator generator
- Arithmetic over $\mathbb{F}_{p^m} \cong \mathbb{F}_p[x]/(f(x))$ for given p, m and $f(x)$
- Support for the following operations:
  - addition
  - multiplication
  - Frobenius map $(a(x)^p \bmod f(x))$
  - inverse Frobenius map $(\sqrt[p]{a(x)} \bmod f(x))$

Future work

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- Side-channel

Thank you for your attention

Questions?