Block Ciphers and Cryptographic Hash Functions

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Part A: Block Ciphers

1. Introduction
   - DES
   - AES
   - Modes of operation & security proofs
2. Differential cryptanalysis
   - Basics
   - Design theories
3. Differential cryptanalysis in practice
4. Linear cryptanalysis, variations on differential cryptanalysis

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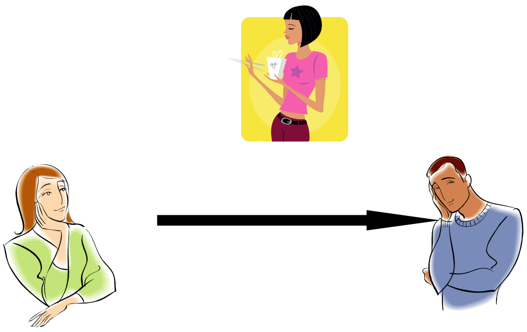
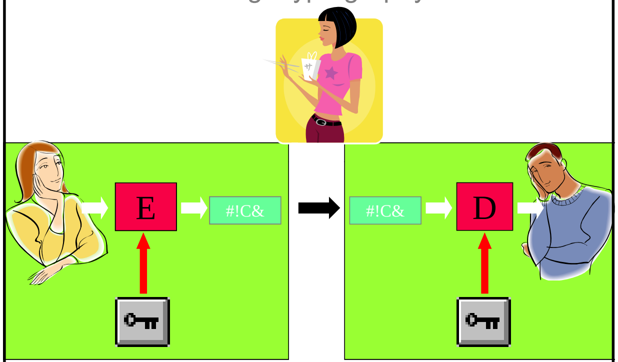
The setting



Using cryptography



Principles

- Kerckhoffs' principle:
  Algorithm is public, except for 1 parameter: the key
- Key generation, distribution, management:
  - Different problem

Goals of Cryptography

- Confidentiality
- Integrity
- Authentication

- Anonymity
- Non-repudiation (origin, delivery)
- Time stamping
- Key escrow

Symmetric cryptography

- Sender and receiver use the same key
  - Or keys that can easily be derived from one another
- Sender and receiver are equivalent
- By far the oldest type of cryptography
- Best performance
- Highest security standards
- Only disadvantage: difficult key management

Practical cryptography

- Short key is used to encrypt long messages
- Perfect secrecy is not possible
- Complexity-theoretic security
  - No satisfactory results thus far
- Practical security
  - Resistance against cryptanalysis
  - "Human ignorance" model

Academic attacks and real attacks

- Academic attack = primitive behaves suboptimal
- Real attack: can be broken in practice
- Example:
  1. Encryption algorithm with 40-bit key
     - Best attack is to try out all 2^{40} keys
     - Practical attack
  2. Encryption algorithm with 256-bit key
     - Key can be recovered with a method that has a complexity equivalent to 2^{200} encryptions
     - Academic attack

Assumptions on the attacker

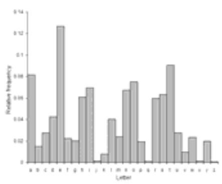
- Ciphertext-only attack
  - Most modern encryption systems are resistant
- Known-plaintext attack
  - Known headers, formatting, ….
  - Can be statistical information
- Chosen-plaintext attack
  - Surprisingly, often quite realistic
- Related-key attack

Simple substitution cipher

- Permutation of the alphabet

A	B	C	…	Z
Q	W	E	…	M

- 26! possibilities (keys)
- Frequency-analysis



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Advanced substitution cipher

- Permutation on *block* of characters

AAAA	AAAB	AAAC	…	ZZZZ
QAQZ	WIJT	ENTO	…	MIHB

- "code book"
- Even more keys
- If blocks large enough, then frequency analysis impossible (infeasible)

Block cipher

- Avoid transport & storage of huge table
- Introduce computation rule to compute table elements:

$$T[X] = f(X, key)$$

- Design 'good' rule f:
  - Secure
  - Efficient

Block cipher formally

- Family of permutations
- Every value of the key selects one permutation
- Block length n: $2^n! \approx 2^{(n-1)2^n}$ permutations
- Key length k: 2^k selectable permutations
- Library of code books

Shannon's view on block cipher security

- Short key
  - Conditional security
- Key determined by equations
  - Derived from message X, ciphertext Y, algorithm B
  - Should be difficult to solve
- Without Key, impossible (*infeasible*) to
  - Decrypt (encrypt)
  - Derive statistical information about the message

Shannon's principles

1. Confusion: equations in the key should be
   - Complicated (non-linear)
   - Involve many variables
2. Diffusion: redundancy in message should be dissipated over large structures in ciphertext

Design principles

- Shannon: product ciphers

$$B = T \circ M \circ S$$

  - M: mixing transformation (known)
  - S, T: simple substitution ciphers (keyed)
- Iterative ciphers:

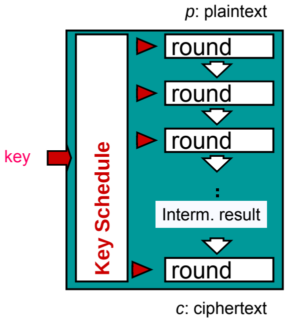
$$B = S_1 \circ M \circ S_2 \circ M \circ S_3 \circ \ldots \circ M \circ S_r$$

  - Round transformation, round: $(S_i \circ M)$
  - Often: $S_i = S \circ \text{AddKey}$

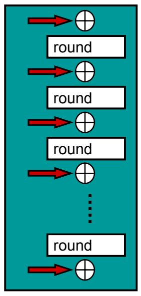
AddKey: key addition

- Injection of key material
  - Addition of key to intermediate variable
  - Use of key-dependent transformations
- Key schedule
  - Input: cipher key
  - Output: round keys

Iterative cipher



Iterative cipher with key addition



'round' doesn't depend on the key

What is nonlinearity?

- Distance to linear functions
  - = how difficult to approximate by a linear function
  - ≠ nonlinear degree
- Example:
  - f(a,b,c,d) = abcd
    - abcd ≈ 0
    - Nonlinearity(f) = d(f,0) = 1/16
  - g(a,b,c,d) = ab + cd
    - Nonlinearity(g) = 6/16

Importance of nonlinearity

- Linear cryptanalysis
  - Linear approximations of the cipher
- Differential cryptanalysis
  - Non-uniformity of first order derivative

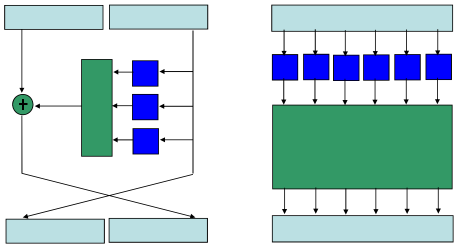
Mixing

- Boolean equations in a small number of variables are always easy to solve
- Mixing needs to ensure strong dependencies between sub-systems
- Easiest to measure for linear transformations (usually)

Practical constraints

- Hardware/software
- Key agility
- Typically
  - Small substitution elements
  - Mixing by means of interconnection

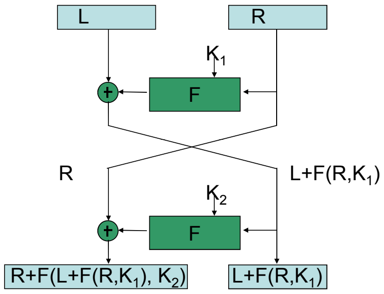
Feistel ciphers and SP-networks



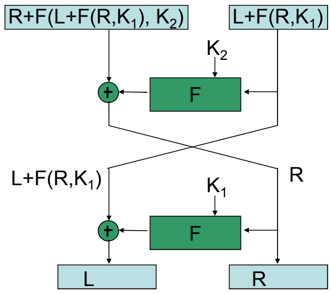
Feistel

- Round transformation is an involution
- Encryption and decryption only differ in the order of the round keys
  - Saves hardware area/code size

Feistel encryption



Feistel decryption



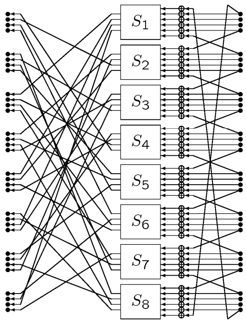
Block cipher research

- Majority of designs uses Feistel structure or uniform structure
- Designs concentrate on selection of nonlinear elements
  - Small elements to reduce cost
  - Connection

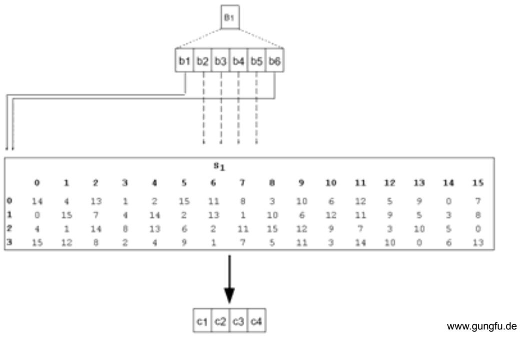
Data Encryption Standard (1977)

- 1970: need for a commercial-grade encryption standard
- 1973-1977: Development of a block cipher DES
  - IBM together with NBS
- Encrypts blocks of 64 bits
- Effective key length of 56 bits
- Structure:
  - Initial bit shuffle
  - 16 iterations of a round transformation (Feistel)
  - Inverse bit shuffle

The DES round function



S-box 1



s_1

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

S-box design criteria

- Surrounded with mystery (“No need to know”)
- Apparently, largest S-box that would make DES fit on a single chip (in 1974)
- S-box input bits
  - 2 row selection bits, 4 column selection bits
  - 2 *middle bits*, 2 times 2 *end bits*
- Every row is a permutation
- End bits are shared between neighbouring S-boxes

S-box design criteria

1. No output bit is close to a linear function of the input
2. Flip one input $\rightarrow$ at least two output bits flip
3. Flip two middle bits $\rightarrow$ at least two output bits flip
4. Flip the first two input bits, but not the last two $\rightarrow$ at least one output bit flips
5. …
6. …

Bit permutation P criteria

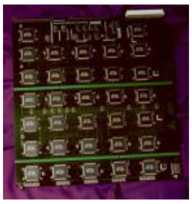
1. For every S-box, two outputs go to middle input bits, and two outputs go to end bits
2. Outputs of every S-box affect 6 S-boxes
3. If output of one S-box affects middle of another S-box, then not vice versa

Rise of the DES

- Design criteria classified
  - Design rationale remained unclear until 1990
- Modifications by NSA
  - Trapdoors?
- Short key length
  - Exhaustive key search
- World-wide adoption: the only commercial standard
- Also used for data authentication mechanisms

Fall of the DES

- Designed for 1970 technology
  - No use of nifty processor features
- 1991, 1993: academic attacks + design of a DES cracker machine
- 1998: exhaustive key search performed in practice (EFF)
- Temporary solution: 3-DES



Multiple encryption

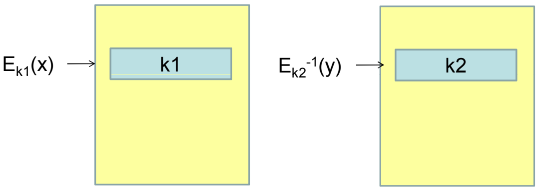
- DES is not a group:
- In general, we can't find a k3 = f(k1,k2) such that

$$E_{k2}(E_{k1}(x)) \equiv E_{k3}(x)$$

- Hence, multiple encryption is not equivalent to single encryption
  - Can be used to increase the key space
- Double encryption is not sufficient

Attack on double encryption

- Known plaintext: $y = E_{k2}(E_{k1}(x))$
- Create two hash tables



- Pairs (k1,k2) at the same address are key candidates
- Attack complexity: 3 times exhaustive search for 1 key

3-DES: triple encryption

- E-E-E or E-D-E
  - E-D-E easier for backwards compatibility
- Triple key or double key: $E_{k1}(E_{k2}(E_{k1}(x)))$
  - Triple key offers more practical security
- Slow
- Alternative: XDES ("triple-key DES")

$$y = k3 + E_{k2}(x + k1)$$

Advanced Encryption Standard

- 1997: public call for submission
- Encrypt blocks of 128 bits
- Key of lengths 128, 192, 256
- To be available royalty-free
- August 1998: first AES conference

Public evaluation

- Only public comments taken into account
- Decisions by NIST, motivated by public reports
- Most analysis done by the public
- NSA had the right to veto NIST's decision

Evaluation criteria

- Security
- Efficiency
- Intellectual Property issues
- Flexibility
- Elegance, ability to prove absence of trapdoors, …

Design trade-off

- Luke O' Connor (IBM):
  "*Most ciphers are secure after sufficiently many rounds*"

- James L. Massey (ETH Zuerich)::
  "*Most ciphers are too slow after sufficiently many rounds*"

Science or Engineering?

- Practical security can be achieved easily if we don't worry about performance

- It is not sufficient to prove that a secure block cipher exists
- We have to construct it

- Design challenge:
  - security AND performance
  - provability

Rijndael

- Based on the dissertations of Joan Daemen (1995) and Vincent Rijmen (1997)

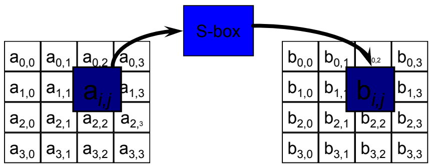
- Not a Feistel cipher (finally!)

- Influenced by experience with chip card based practical systems

Rijndael: Iterated Block Cipher

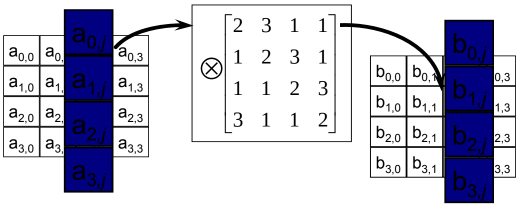
- 10/12/14 times applying the same round transformation
- Uniform round transformation
- Composed of 4 steps, each its own purpose:
  - SubBytes: non-linearity
  - ShiftRows: inter-column diffusion
  - MixColumns: inter-byte diffusion within columns
  - AddRoundKey

Round step 1: SubBytes



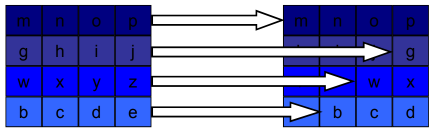
- Bytes are transformed by invertible S-box.
- One S-box (lookup table) for complete cipher:
  - High non-linearity: multiplicative inverse in $GF(2^8)$
  - Complex algebraic expression: additional linear transformation

Round step 3: MixColumns



- Columns transformed by matrix over GF(2^8)
- High intra-column diffusion:
  - based on theory of error-correcting (MDS) codes

Round step 2: ShiftRows

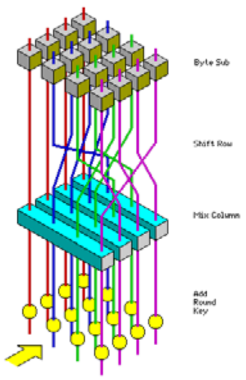


- Rows are shifted over 4 different offsets
- High diffusion over multiple rounds:
  - Interaction with MixColumns
  - Bits flip in minimum 25 active S-boxes per 4 rounds

Round step 4: Key addition

$$\begin{bmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} + \begin{bmatrix} k_{0,0} & k_{0,1} & k_{0,2} & k_{0,3} \\ k_{1,0} & k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,0} & k_{2,1} & k_{2,2} & k_{2,3} \\ k_{3,0} & k_{3,1} & k_{3,2} & k_{3,3} \end{bmatrix} = \begin{bmatrix} b_{0,0} & b_{0,1} & b_{0,2} & b_{0,3} \\ b_{1,0} & b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,0} & b_{2,1} & b_{2,2} & b_{2,3} \\ b_{3,0} & b_{3,1} & b_{3,2} & b_{3,3} \end{bmatrix}$$

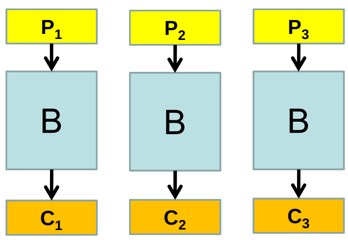
- Makes round function key-dependent
- Round keys derived in a simple way from the master key



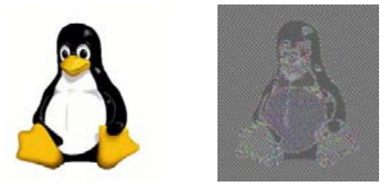
Modes of operation

- How to encrypt data that is not exactly one block?
  - Integer number of blocks
  - Fractions of blocks
- Using block ciphers for other goals than encryption
  - MACing
  - Hashing
- Consequence of popularity of the DES

Electronic Code Book

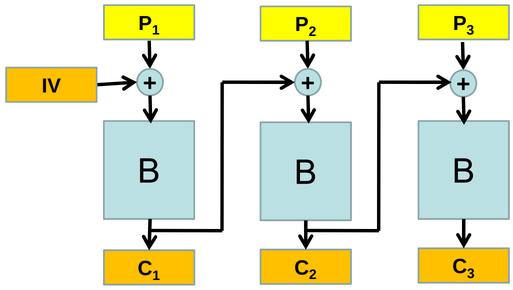


Problem of ECB



Source: Wikipedia

Cipher Block Chaining



Properties

- Patterns are hidden
- Even repeated encryption of the same message not detectable (by changing IV)
- Last ciphertext block depends on all plaintext blocks
- Not true for decryption direction: each plaintext block depends on only two ciphertext blocks
- Favourite encryption mode (definitely in the past)

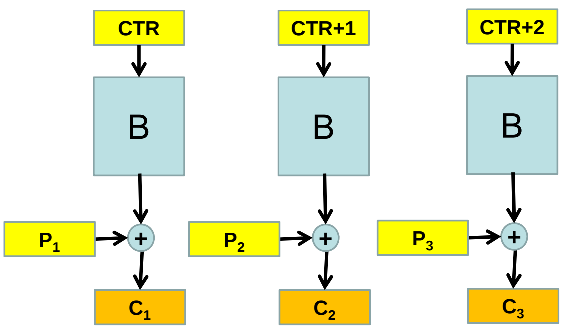
Birthday attack

- Encrypt $2^{n/2}$ blocks under the same key
- With high probability:

$$\exists\, i, j \text{ such that } C_i = C_j$$
$$\Updownarrow$$
$$C_{i-1} \oplus P_i = C_{j-1} \oplus P_j$$
$$\Updownarrow$$
$$P_i \oplus P_j = C_{i-1} \oplus C_{j-1}$$

- Information on plaintext revealed
- Encrypting slightly more blocks leads to many more collisions
- Main reason why AES has block length 128

Counter Mode



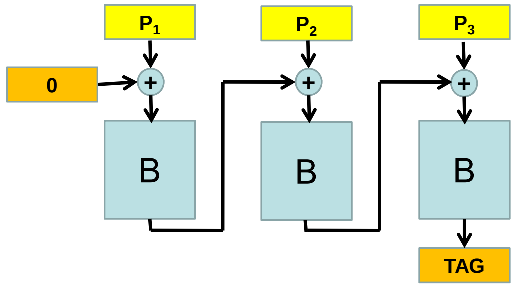
Properties

- Counter should start at values sufficiently far away from one another
  - Never same inputs to block cipher
- Parallel
  - Pipelining
  - Random access (hard disks)
- Block cipher is used to build a stream cipher

Message Authentication Code (MAC)

- Cryptographic check sum
- Allows to detect malicious modifications to messages
- Sender and receiver use the same key
  - Not a digital signature

CBC-MAC (Simple MAC, S-MAC)



Authenticated encryption modes

- Combine encryption and authentication
- Less errors
  - Order of encryption and authentication
  - Different keys or the same
- Faster
  - One pass over the data
  - Not true for unpatented schemes
- Security proofs

Security proofs

- \`But that’s not security,’ said Alice, \`security means something else.’
- \`Security means what I choose it to mean,’ said the queen.

Alice in Wonderland

Security proofs for modes

- Concrete
  - For one or more given block ciphers
- Standard model
  - Block cipher is a Pseudo-Random Permutation (PRP)
- Random Oracle Model – Ideal cipher model

Pseudo-Random Permutation (PRP)

- *Function indistinguishable from random permutation*
- There are $2^n!$ permutations from n bits to n bits
- Denote by R the set of all n-bit permutations
- *Random permutation*: randomly selected element of R

Further definition:

- *Oracle*: black box: for each input, it gives the output of the function it implements

Distinguishing

Game: for $r \in R$, $f \in F$

When given two oracles, one for r, one for f

Say which is which

- Average probability of success – 0.5 = *Advantage*
- Advantage depends on
  - Number of oracle accesses (queries)
  - Computational power (usually: not limited)
  - Size n

Indistinguishable

- We look at what happens when n grows

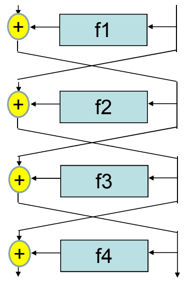
- Advantage = f(q,n)
- A primitive is called *indistinguishable from random* if
  - f decreases as an exponential function of n
  - Even if q grows as a polynomial function of n

Block cipher as Pseudo-Random Permutation

- Block cipher is family of permutations
  - One for each key
- We know constructions to build block ciphers that are PRPs
  - Luby-Rackoff

- Security proofs for applications: if the block cipher is a PRP, then …

Luby-Rackoff construction



If f1, f2, f3, f4: are pseudo-random functions, then this is a PRP

Note that we can't really build this in practice

PRP

- A PRP can have:
  - Weak keys
  - Equivalent keys
  - Output the key upon receipt of a special plaintext

Because the model considers only the 'average case'

(On average, pedestrians walk in the middle of the road)

- A PRP can further have
  - Weaknesses only apparent if you consider more keys (*related keys*)

Because the model doesn't consider this

Ideal Cipher Model

- The attacker is not allowed to look at the block cipher
- Should help to concentrate on the security of the *mode*

- Argument pro
  - Allows to prove security where the standard model doesn't
    - Block cipher based hash function
    - Anything where key input is not random

- Argument contra
  - 'prove security' means here: define security as the property that you can prove

Use of security proofs

- Definitely, don't use a mode of operation proven insecure
- Is it better to have a proof of security than to have no proof?
  - Yes, if everything else is equal ☺
- We don't know how to build block ciphers that can be proven to be PRP, are efficient and use a short key
- There is no idea how to measure whether a block cipher is close to ideal

Secure mode of operation

- Submit q queries of length n, 2n, 3n, …
- Try to distinguish
  - Mode M with block cipher replaced by ideal cipher
  - Large ideal cipher (with variable block length)
- Advantage = f(q,n)
- Mode M is secure if
  - f decreases as an exponential function of n
  - Even if q grows as a polynomial function of n

ECB is insecure

- Submit (P,P)
- Oracle answers (C_1, C_2)
- For ECB: $C_1 = C_2$ always
- For ideal cipher with block length 2n:

  $C_1 \neq C_2$ with probability $1 - 2^{-n}$

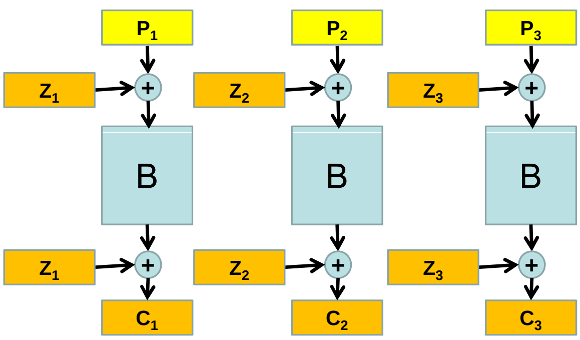
CBC is secure

- But need to use a new, unpredictable IV every time
- Otherwise, submit P_1 and (P_1, P_2)
- What about the birthday attack?
  - q grows exponentially
  - Not allowed

CBC-MAC is secure

- But only if all messages have the same length!
- Let $T_1 = MAC(X_1)$, $T_2 = MAC(T_1)$
- Then $MAC(X_1, 0) = T_2$
- (Can be fixed easily)

Offset Code Book (OCB)

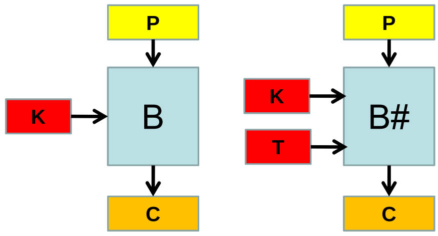


OCB start and stop

- *Whitening values* Z_i
  - γ_i: gray code counter
  - $Z_i = \gamma_i \times E[0] + E[Nonce + E[0]]$
- Final values (tags)
  - $C_{n+1} = E[Length(P) + E[0] \times x^{-1} + Z_{n+1}] + Length(P)$
  - $C_{n+2} = E[\Sigma_i P_i + Z_{n+1}]$
- Provably secure against
  - Distinguishing attacks
  - Forgery attacks

Tweakable block cipher

- Idea: introduce additional variability: the tweak parameter
  - Known to the attacker



Provable security

- If a secure tweakable block cipher exists, then also a secure block cipher exists (obviously)
- If a secure block cipher exists, then also constructions for secure tweakable block ciphers exist
- Tweakable block ciphers simplify (proofs of) modes
  - OCB is close to ECB with tweakable block cipher

Conclusion

- Practical block ciphers, DES, AES
- Shannon's ideas on practical designs
- Modes of operation
- (Security) proofs