B-it **IPEC Winter 2009**

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Exercise sheet 1: Preliminaries

I Elementary probabilities

I.1 Exercise 1

According to statistics, a pedestrian walking on the emergency stripe of a high way has a probability of 8% to be hit by a car every minute. Mr X decides to do the following stupid bet with his friend: if he manages to walk 15 minutes on the emergency stripe without being victim of an accident, his friend owes him 200 Euros.

What is the probability the Mr X gets the money?

I.2 Exercise 2

We throw many times a fair die with 6 faces. How many throws should one make in order to obtain an as with probability 50% ? 99% ?

I.3 Exercise 3

A professor decides to do quickly the following oral exam of "Probabilities". The student is authorized to distribute 4 balls, 2 black and white, in two urns. The professor chooses randomly one urn and extracts a ball. If the ball is black, the student passes the exam.

How would you distribute the balls?

II Conditional probabilities

A student, Peter, usually goes out on Friday evening, with a probability of 2/3. But, on that day, there is a math exam on the following day. We assume that Peter passes the exam with a probability of 3/4 if he did not go out on the eve, and only 1/2 otherwise.

Given that Peter did not pass the exam, what is the probability of having gone out on the exam's eve?

III Discrete random variables

To determine the final mark, a professor proceeds as follows: he throws two dice, and considers the smallest value, say X. He defines then a random variable N to be $3X$.

Compute the expectation and variance of the random variable N.

IV Axiomatic definition of entropy

A measure of the "uncertainty" of an event with probability p, can be modeled using a function $h(p) \geq 0$ satisfying the following axioms:

- $h(p)$ is a function decreasing in p.
- $h(pq) = h(p) + h(q)$ for all p, q.

1. Justify heuristically those axioms.
2. Show that $h(1) = 0$ and justify this result.

   We consider two probabilities p and q and an integer $m > 0$ sufficiently big.
3. Show that one can find an integer $n \geq 0$ such that
$$q^{n+1} \leq p^m \leq q^n.$$
4. Deduce the following relations
$$\frac{n}{m} \leq \frac{h(p)}{h(q)} \leq \frac{n}{m} + \frac{1}{m}$$
   and
$$\frac{n}{m} \leq \frac{h(1/p)}{h(1/q)} \leq \frac{n}{m} + \frac{1}{m}$$
5. Conclude the general form of $h(p)$.
6. Justify by this approach the definition of the entropy of a discrete random variable as a measure of the average uncertainty.