The Fuzzy Vault for fingerprints is Vulnerable to Brute Force Attack, the Collusion Attack

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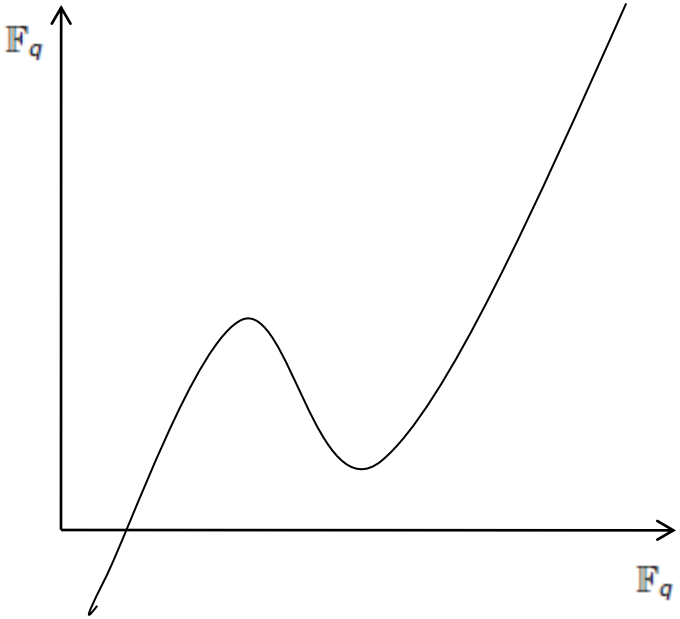
Overview

- Motivation
- Short reminder of the Fuzzy Vault for fingerprints
- Vulnerability to Brute Force Attack
- The Collusion Attack
- Solution to increase security: Quiz

Motivation

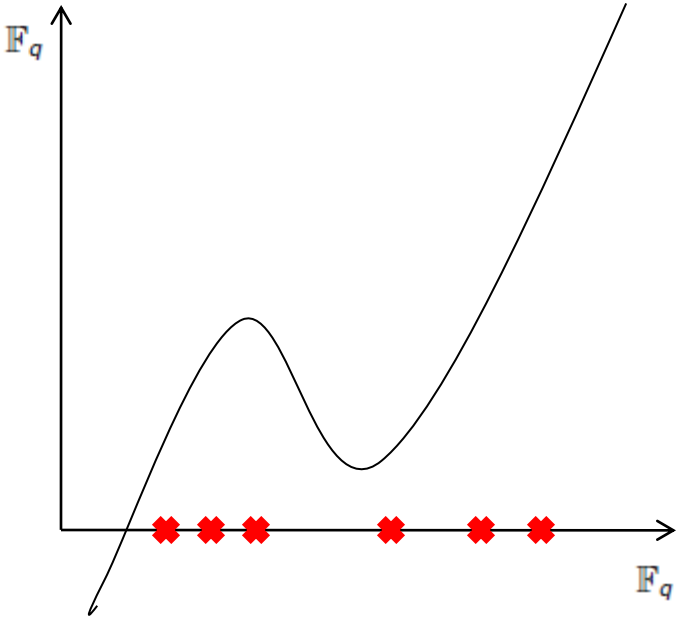
- Fuzzy Vault scheme is an encryption scheme, which can tolerate errors in the keys (e.g. fingerprint scanning issues)
- Since an attacker could get access to the message of a vault, he will be able to obtain the key by brute force attack
- If multiple vaults are known, it is even easier to obtain the key by just comparing x-values
- The Fuzzy Vault scheme can be improved by implementing additional features which increase Security

Reminder: Fuzzy Vault (encoding)



- Polynomial $f \in \mathbb{F}_q[X]$ of degree k identifies a secret S

Reminder: Fuzzy Vault (encoding)



- Polynomial $f \in \mathbb{F}_q[X]$ of degree k identifies a secret S
- hiding f using $t > k$ minutiae locations, the locking set $\mathcal{L}$

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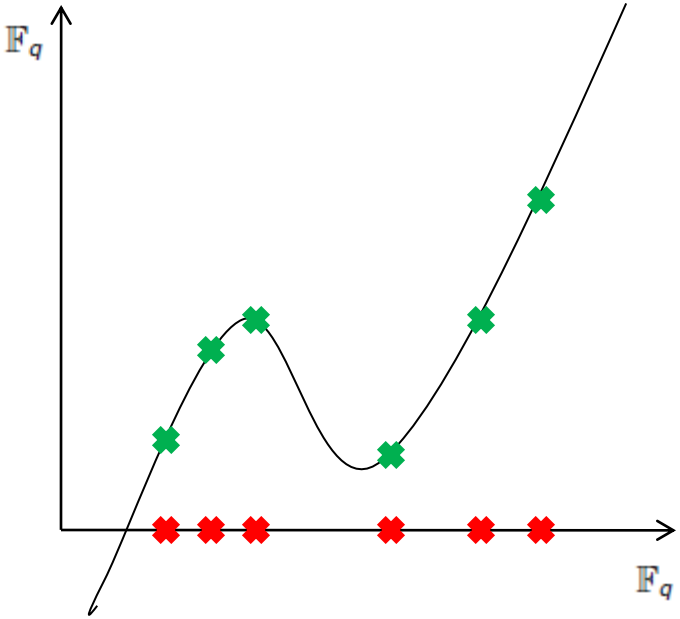
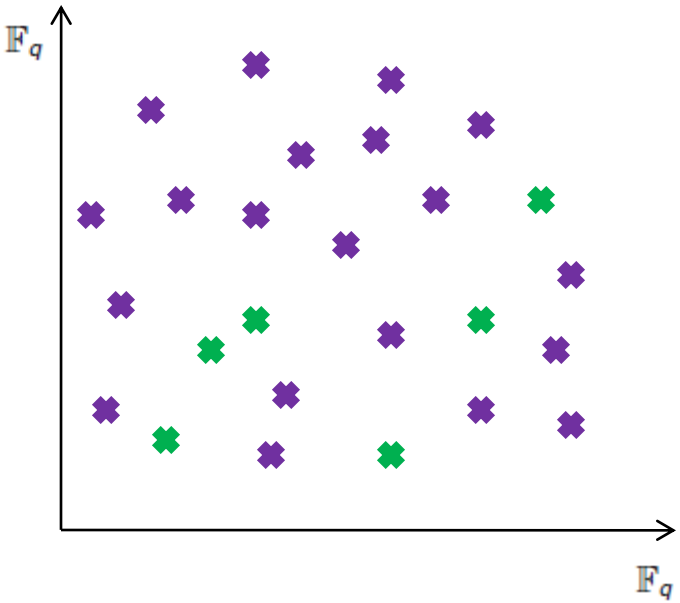


- Polynomial $f \in \mathbb{F}_q[X]$ of degree k identifies a secret S
- hiding f using $t > k$ minutiae locations, the locking set $\mathcal{L}$
- Evaluating the locking set $\mathcal{L}$ with f to the genuine set $\mathcal{G}$

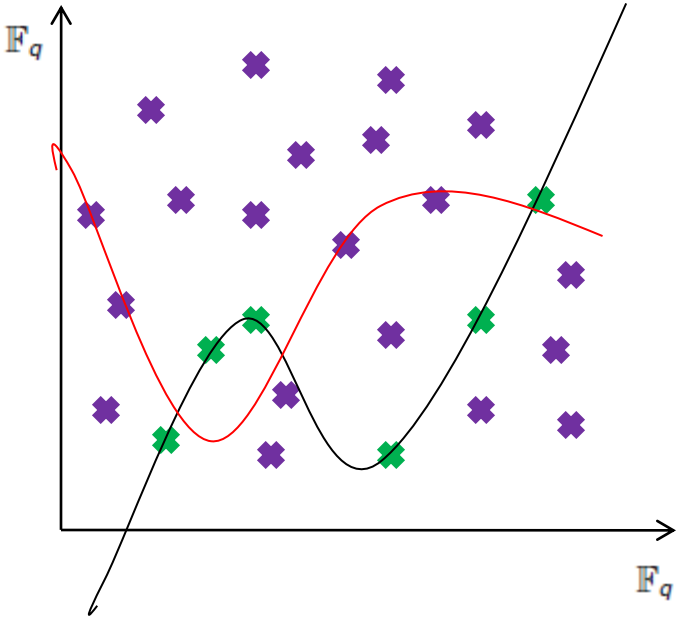
Reminder: Fuzzy Vault (encoding)



- Generate random Chaff Points to hide the genuine set $\mathcal{G}$
- The Vault is now:

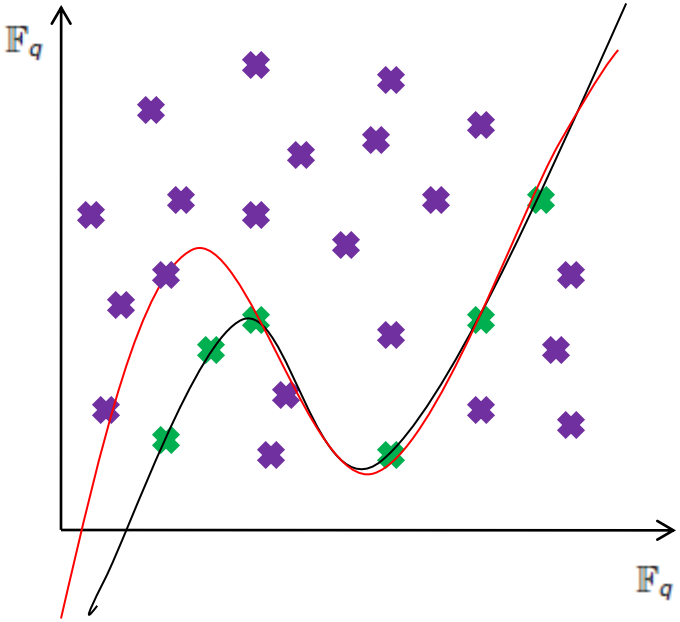
$$\mathcal{V} = \mathcal{V}(k, t, r, \mathbb{F}_q) = \mathcal{G} \cup \mathcal{C}$$

Reminder: Fuzzy Vault (open the Vault)



- $K + 1$ genuine points must be known to open the vault and to compute its interpolating polynomial
- False points result in a completely different curve.

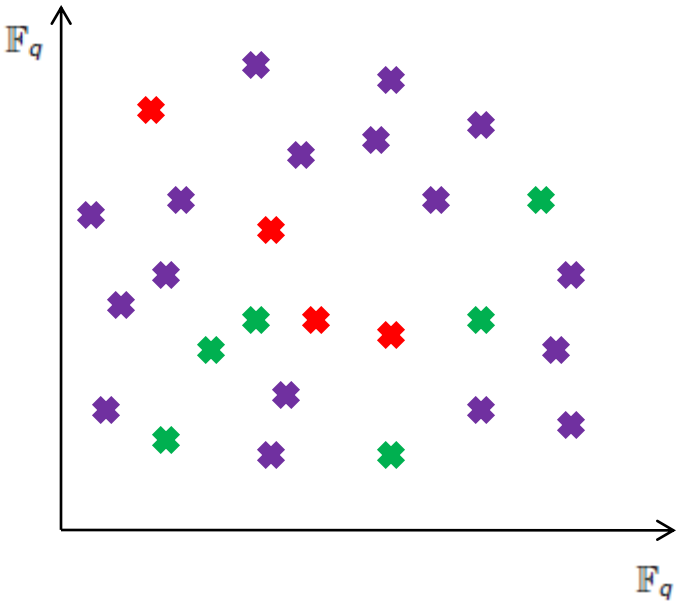
Reminder: Fuzzy Vault (decode)



- A second genuine set $\mathcal{G}'$ of the same fingerprint might not be identical to the original genuine set (distortion, rotation)
- If the distance is not to far, an error correction algorithm makes a successful decoding possible

Vulnerability to Brute Force Attack

$$\mathcal{V} = \mathcal{V}(k, t, r, \mathbb{F}_q) = \mathcal{G} \cup \mathcal{C}$$



- Choose t + 1 distinct points at random

Vulnerability to Brute Force Attack

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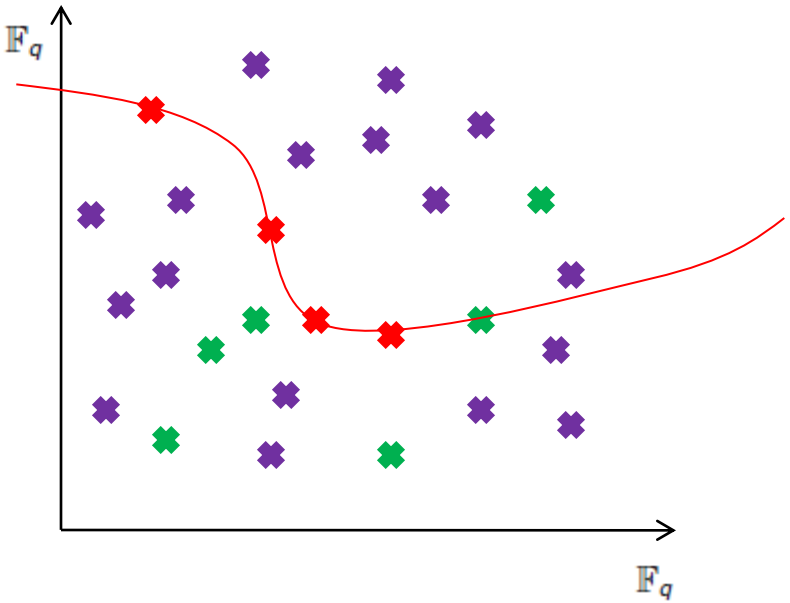

- Choose $t + 1$ distinct points at random
- Compute $f \in \mathbb{F}_q[X]$ by interpolating the chosen points.

Vulnerability to Brute Force Attack

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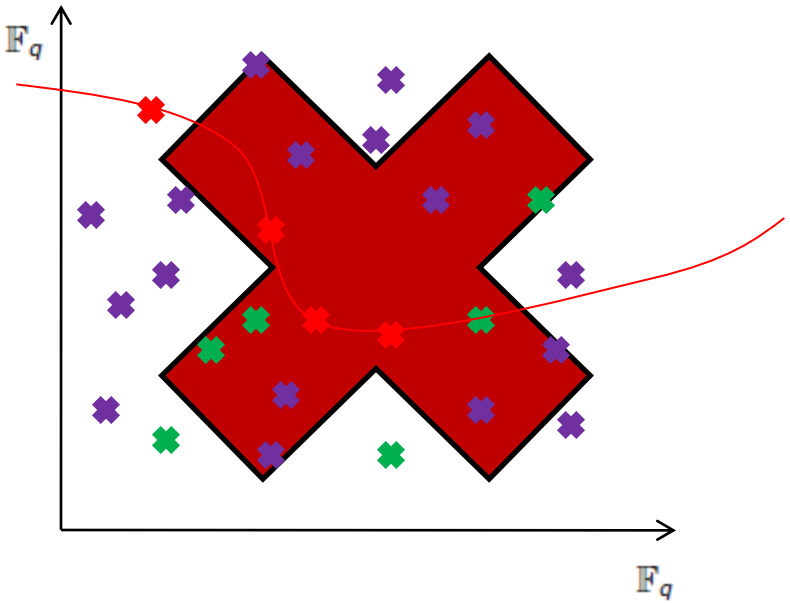

- Choose $t + 1$ distinct points at random
- Compute $f \in \mathbb{F}_q[X]$ by interpolating the chosen points.
- If the graph of f contains t vault points, output f, otherwise start all over again.

Brute Force Attack will finally lead to…

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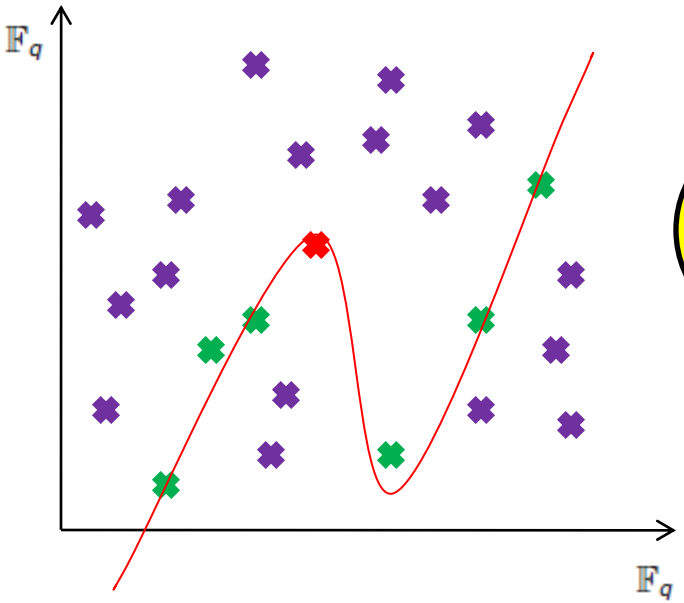

- Choose $t + 1$ distinct points at random
- Compute $f \in \mathbb{F}_q[X]$ by interpolating the chosen points.
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Vulnerability to Brute Force Attack

- Given the Vault $\mathcal{V} = \mathcal{V}(k, t, r, \mathbb{F}_q) = \mathcal{G} \cup \mathcal{C}$, the probability **P** that a brute force attack by randomly chosing *k* + 1 vault points an interpolate them will result in the correct *f* is:

$$1/\mathbf{P} = \frac{\binom{r}{k+1}}{\binom{t}{k+1}} < 1.1 \cdot (r/t)^{k+1}, \quad \text{for } r > t > 5.$$

- Computing the interpolation polynomial *f* can be done in $6.5 \cdot k \cdot \log^2(k) + O(k)$ operations.
- $\sum_{i=1}^{t-k-1} (1/q)^i$ is the probability that *t* points lie on the graph of the polynomial *f*

Vulnerability to Brute Force Attack

- Complexity: An Intruder with the knowledge of an intercepted ν can recover the secret *S* in $C \cdot (r/t)^k$ operations, where *C < 8rk.*

<u>*Proof:*</u> Since wie have $1/\mathbf{P} = \frac{\binom{r}{k+1}}{\binom{t}{k+1}} < 1.1 \cdot (r/t)^{k+1}, \quad \text{for } r > t > 5.$

an attacker has to compute the interpolated polynomial *f* to his randomly chosen vault points each try, which can be done in *6.5*log^2(k) =:* ***K***.

Vulnerability to Brute Force Attack

So computing all interpolated polynomials can be done in *< 7.2*k*log^2(k)*(r/t)^k* operations.

Search a point *(U,W) ϵ* $\mathcal{V} \setminus \mathcal{T}$ such that g(U)=W.

This requires r/K interpolations. When no point is found, discard $\mathcal{T}$. If it was not discarded, search for a further point, which has this properties. This step has the probability 1/q. If a point has been found, add it to $\mathcal{T}$, otherwise discard it.

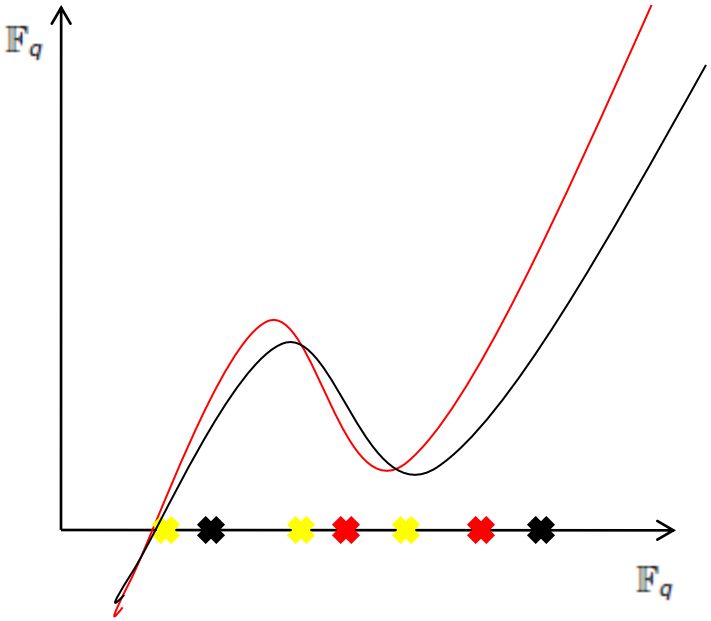
This continues until the attack is finished. Adding up all Numbers of those steps with weights given by the probability of occurrence, one finds:

$$R < 7.2 \cdot (r/t)^k \cdot k \cdot K \cdot r \cdot \sum_{j=0}^{\infty} (1/q)^j = 7.2 \cdot (r/t)^k \cdot k \cdot K \frac{rq}{K(q-1)} < 8.0 \cdot (rk) \cdot (r/t)^k$$

Factors that influence the complexity of a b.f.a.

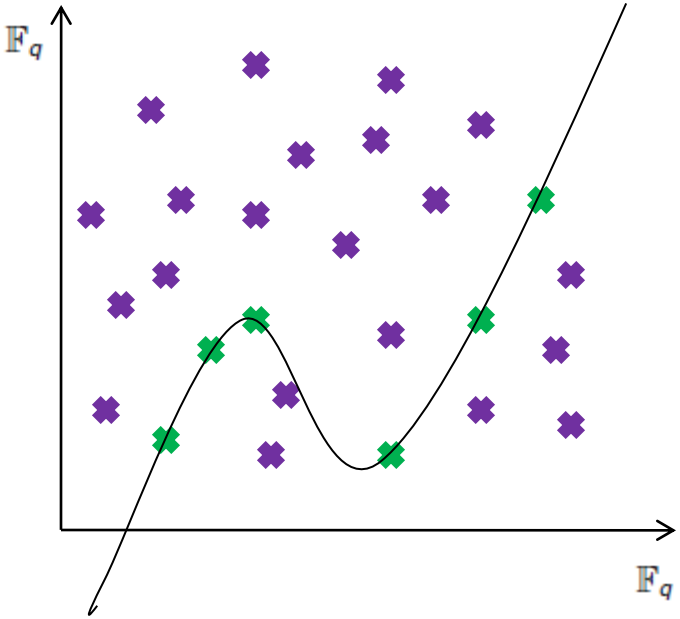
- Restricting the region of interest
  - irrelevant, when minutiae uniformly distributet, because r and t are scaled by the same factor, r/t and the complexity remain unchanged.
- Increasing *k*
  - complexity grows
  - requires large unlocking sets
  - might be a problem for fingerprint scanners
- Increasing #Chaffpoints
  - complexity grows
  - smaller distances between points in the vault
  - can compromise error corrections during unlocking
- Reducing *t* in the genuine List
  - Lesser points of interest
  - Could reduce the size of the unlocking set below the required minimum

The Collusion Attack



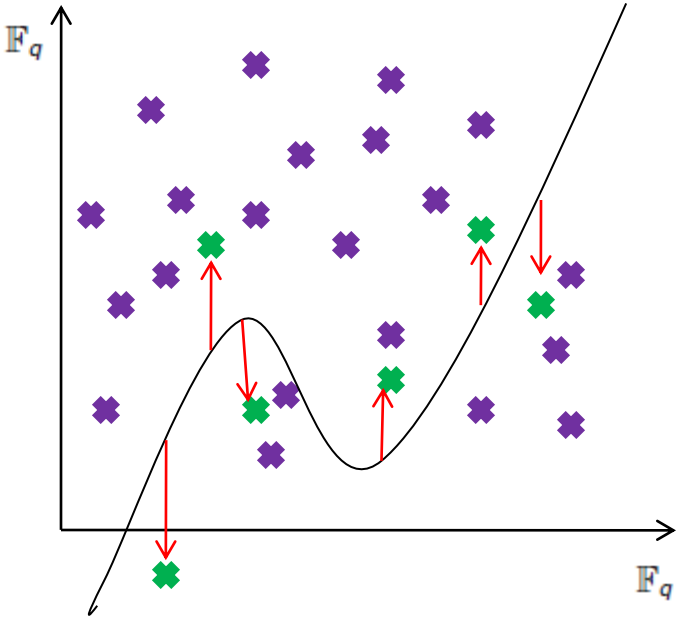
- Possible Attack, if an attacker is aware of more than one Fuzzy Vault, e.g. a few smartcards carrying the same biometric information (fingerprint).
- Attacker can compare the x-coordinates of both templates.
- Some may be different because of different scanners, but a few might be identical.
- All found identical x-coordinates are stored during this attack, the others are recognized as Chaffpoints and removed. The remaining points result in the effective vault.
- With these x-coordinates, the polynomial could be interpolated to unlock the vault, because it could be successfully authentificated after error correction
- More known Vaults meens higher probability to find identical x-coordinates.

Fuzzy Vault with Quiz



- Additional minutiae information increases security
- Def.:Let $(X, Y) \in \mathbb{F}_q^2$ a genuine vault point and α the orientation with granularity π/n, n small.
- α is used to change Y-Coordinates of the genuine vault points.

Fuzzy Vault with Quiz



- Choose random β
- *j* encodes transformation

  $T : \mathbb{F}_q \rightarrow \mathbb{F}_q$
- Compute *j* so that *j**π/n = α – β mod π.
- Now genuine points are transformed to the pattern (X,Y‘, β), with Y‘=T(Y).
- Chaffpoints are treated the same way

More improvement ideas…

- Using more fingers
- Non-random Chaff points
  - Guaranteed distances between vault points
  - Easier to maximize the number of points in the Field
- CRC
  - Additional checksum makes decoding easier