29.5 years of Maple: how many of the design principles of the system paid dividends

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Abstract

Most of the original literature about Maple described it as a "compact and efficient computer algebra system". It was partly our goal to be able to run in small desktop computers and even on a pocket computer (the term "pocket symbolic" was also used). This talk will concentrate on four aspects of the early design that went in this direction and were the cornerstones of the design. These are the use of the language C, the $S^2T$ measure of complexity, option remember and its implication which is the unique representation of subexpression and the systematic elimination of quadratic algorithms.
The $S^2T$ measure comes from lower bounds of computational problems: e.g. when $S$ is the auxiliary storage available and $T$ is the time used, every algorithm must use $S^2T = \Omega(n^2)$.
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Usually there are severe restrictions on the computational model, for example, the input is available on a Turing machine tape.
The $S^2T$ measure (II)

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That is, an algorithm which performs better under this measure is usually a better algorithm, not just a fluke due to operating on a different complexity space.
The $S^2T$ measure (III)
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In any case, it is good to have a measure so that algorithms can be benchmarked and the best can be selected. It is the basis of scientific software development.
The $S^2T$ measure (the paper)

A TIME-SPACE TRADEOFF FOR SORTING ON NON-OBLIVIOUS MACHINES

Allan Borodin
Michael J. Fischer
David G. Kirkpatrick
Nancy A. Lynch
Martin Tompa

ABSTRACT

A model of computation is introduced which permits the analysis of both the time and space requirements of non-oblivious programs. Using this model, it is demonstrated that any algorithm for sorting $n$ inputs which is based on comparisons of individual inputs requires time-space product proportional to $n^2$. Uniform and non-uniform sorting algorithms are presented which show that this lower bound is nearly tight.

1. Motivation and Contraposition to Previous Research

The traditional approach to studying the complexity of a problem has been to examine the amount of some single resource (usually time or space) required to perform the computation. In an effort to better understand the complexity of certain problems, recent attention has been focused on examining the tradeoff between the required time and space. This paper adopts the latter strategy in order to pursue the complexity of sorting.

values. In order to truly understand the complexity of sorting, then, a model which admits non-oblivious algorithms should be adopted. Toward this end, Munro and Paterson considered non-oblivious sorting algorithms which use auxiliary registers to store selected inputs and can access other inputs only through successive passes over all the inputs. Although they count only data space (i.e., number of auxiliary registers used), the authors make it clear that "control space" (used, for instance, to remember which inputs to fetch into registers on a given pass) is also an issue in upper bounds. To sort $n$ inputs within their model, they demonstrate that the product of the number of registers and the number of passes is $\Theta(n)$. Since each pass requires $n$ moves of the input head, their result might be interpreted as a lower bound of $\Omega(n^2)$ on the product of time and data space. Adopting Cobham's model Tompa in fact demonstrated a similar tradeoff for sorting on any general string-processing model, exploiting only the restriction of "tape input" (i.e., the input head can move at most one symbol left or right in one step).
The $S^2T$ measure (the paper)

This is the paper that I had in mind:

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The *ST* surprise
The big surprise is that this paper proves, that for sorting, $ST = \Omega(n^2)$.
The $O(n^2)$ hidden bugs

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These bugs go often unnoticed until a production-type problem is submitted.
The $O(n^2)$ hidden bugs (II)
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```latex
res := NULL;
for i to n do res := res, f(i) od;
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res = New(EXPSEQ);
for( i=0; i<n; i++ )
    res = append(res,f(i));
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For those in the internal mailing groups, it is still common to see reports of $O(n^2)$ new bugs.
The $O(n^2)$ hidden bugs (IV)
The only answer that I have to this problem is to create a "Quadratic Police"
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“Cubic or higher Army” and “Exponential nuclear deterrent”
“Option remember” is the term that we use to describe the ability of a function of remembering previous arguments/results and avoid/save computation.
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F := proc( n::integer )
option remember;
if n < 2 then n else F(n-1)+F(n-2) fi
end:
Remember and unique representation (II)

diff( tan(x), x$100 );
It may change an algorithm from exponential to linear in time (and/or space) required.
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(At one point I wrote a program to create the largest expression ever (a Guiness-type record). This expression was so large that any linear function would never return, only remembering functions had a chance).
The rationale for remembering is that computer algebra (manipulated mathematical expressions) contain highly repetitive parts.
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All expressions are simplified and unified recursively. Duplicates are discarded.
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But, there is no question in my mind that it makes for the largest space/time economy in the early Maple.
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It is not a surprise, in this context, that the heuristic GCD algorithm became a leading example of several similar algorithms and a cornerstone of Maple’s efficiency.
How many times did I hear: “why do you bother about memory, memory is cheaper every day”? 
Memory and Ghz are cheap

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Answer: as many as #(fools) × #(encounters)
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A system which uses memory wisely will always be ahead of one who doesn’t. (paging, garbage collection, compaction, etc. will also cost time)
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The architecture of Maple was strongly influenced by the language (B and later very simple C). The language itself was influenced by Algol68.
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Once I asked one of the top system builders of the time: what do I get from Lisp? After an extremely long pause the answer was: “garbage collection for free”.

We do not need to defend this decision, just observe the language in which the current top CA systems are written.
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We did not think very highly of this remark at the time. Maybe this was a very good compliment that we did not appreciate enough.
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The speed of starting is a combination of various aspects: small kernel, load-on-demand library, efficient language, among others.
Natural selection, if applicable to software, shows that some subset of these features is very good, as Maple survived 29.5 years
the END