0. Repetition sheet

Exercise 0.1 (High powers). Compute $3^{98765432101}$ in $\mathbb{Z}_{101}$.

Exercise 0.2 (Touching $\mathbb{F}_4$). Consider polynomials of degree less than 2 over the field $\mathbb{F}_2$. Define addition and multiplication of them modulo the polynomial $X^2 + X + 1$.

(i) Write down the complete list of elements.
(ii) Write down the addition table.
(iii) Write down the multiplication table.

We can now consider polynomials over $\mathbb{F}_4$: $T^2 + T + 1$ is such a polynomial. Factor it (over $\mathbb{F}_4$).

Exercise 0.3 (Computing in $\mathbb{F}_{256}$). Let $M$ be your student id. Let $a = M \mod 256$, $b = (M \div 256) \mod 256$, and $c = (a + b) \mod 256$

Now interpret $a$, $b$ and $c$ as elements of $\mathbb{F}_{256}$. Compute in $\mathbb{F}_{256}$

(i) $a + b$ (Attention! Usually the result will not be $c$!),
(ii) $a \cdot b$, and
(iii) $1/a$ (or $1/b$ in case $a = 0$).

Note: If $x = x_1 \cdot 256 + x_0$ with $0 \leq x_0 < 256$, then $x \div 256 = x_1$ and $x \mod 256 = x_0$.

Exercise 0.4 (Computing inverses). If possible compute the inverse

(i) ... of 89 in the ring $\mathbb{Z}_{101}$,
(ii) ... of 42 in the ring $\mathbb{Z}_{1001}$.

Give a proof if no inverse exists.