Bonn-Aachen International Center for Information Technology
Cryptography and Game Theory — Raekow, Ziegler
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1 Game Theory - Introduction

The birth date of the Game Theory is 1944 year. Joh von Neumann and Oskar Horgustern have published the first book on this topic "Theory of Games and Economics Behaviour". In all game models which will be discussed in the lectures from now on we will assume the following behavior for all players:

- rational reaction
- maximize your profit with every action

In 1994 year John Nash, Jr. John Haarsangi and Reinhard Selten get a nobel prize.

1.1 Game - What is a Game ?

- model of interaction between two or more players (decision makers)
- listing all possible actions
- evaluating possible outcomes

List of al different type of games which will be discussed

- Strategic games
- Extensive form games
- Extensive form games with incomplete information

2 Strategic games

In these game the player decides and executes his action once. All actions are executed simultaneously. Usually such games are called 'one shot' games.

Definition 1 *A strategic game is a triple* $\mathfrak{G} = (N, (A_i)_{i \in N}, (N_i)_{i \in N})$ *consisting of*

- *a finite set* $N = \{1, ..., n\}$ *of players*
- *for each player* i*, a set of actions* A_i*(finite)*
- *for each paler* i*, a utility function* $u_i : A_1 \times A_2 \times ... \to \mathbb{R}$

A vector $a = (a_1, ..., a_n) \in A = A_1 \times A_2 \times ... A_n$ is called an outcome of $\mathfrak{G}$
Strategic games are also known as:

- games in shorter form
- games in matrix form
- one-shot games

Definition 2 *We call a game finite if the cardinality of all sets A_i is finite.*

Assumptions:

- players' only objective is to maximize their payoff. They act rationally.
- all rules (i.e. available actions) are publicly known
- utilities are public knowledge
- there are no computational restrictions
- all actions are taken simultaneously

Representation by matrix:

P1 \ P2	L	R
T	$u_1(T, L)\ u_2(T, L)$	$u_1(T, L)\ u_2(T, L)$
B	$u_1(T, L)\ u_2(T, L)$	$u_1(T, L)\ u_2(T, L)$

2.1 Solutions in pure strategies

Example: BoS

boy \ girl	Baseball	Softball
Baseball	$2, 1_{NE}$	0,0
Softball	0,0	$1, 2_{NE}$

Definition 3 *A Nash equilibrium (NE) of a strategic game is a profile $a^{\grave{}} \in A$ of actions, s.t. for every $i \in N$ and $a_i \in A_i, u_i(a_i^*, a_{-i}^*) \geq u_i(a_i', a_i^*), a^{\grave{}} = (a_1^*, ..., a_1^*)$*

Example: Prisonor's Dillema

P1 \ P2	Don't Confess	Confess
Don't Confess	$-1, -1$	$-4, 0$
Confess	$0, -4$	$-3, -3_{NE}$

Example: Hawk - Dove (Chicken)

P1 \ P2	Hawk	Dove
Hawk	$-3, -3$	$1, -2_{NE}$
Dove	$-2, 1_{NE}$	0,0

Example: Matching Pennies - no NE!

P1 \ P2	Head	Tail
Head	$1, -1$	$-1, 1$
Tail	$-1, 1$	$1, -1$

Example: Second price auction: (Vickrey action) N players are valuing an object $v_1 \geq v_2 \geq ... \geq v_n$. The highest bid wins the object. (When several player has the same price, the one with the lowest index wins.)

Claim: Bidding v_i is a dominant strategy (No-brainer)

Definition 4 *An action a_i is dominant strategy, if $\forall a_{-i} \in A_{-i}$ $a' \in A_i, u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i})$*

Proof: b_i bid of i-th player

[some pictures]