

Cryptography, winter 2015/16

MICHAEL NÜSKEN, SIMON SCHNEIDER

2. Exercise sheet

Hand in solutions until Friday, 13 November 2015, 23:59

Exercise 2.1 (Break Vigenère).

(8+4 points)

The following text is a Vigenère encryption.

ZLL PIEZ UBIAFOSU AIE GWRIL RUV ALM ROVZX BUSI, OETR OR QIAF, UR
TEG 21, 2061, MZ E AMUQ CLLR PGSEUMBK LMYWB EZIWTMP ORAS BTK
PPKPF. ZLL UCQYXPSV OGQL EJAAX HW I DKWBPB AL E MMDQ-JSSPID
HIA SDQX LPKPNGPSW, IZJ MA LIBVIUIL FNMZ AIK:

GPLBIZJIY ELQRP HRL NKVAVIY RYWS D IKVL XEA UJ ALM RGMALNGR
EAXMZJEUXA AL QBPBUBEJ. EA IKPS EA MTC OYUMT FLMVSY GUYTP,
ZLLC SZKA DLIF REF FMTORK XPQ ISSH, KXOGRMVS, LPHWPUTK MEKQ
– SMSIA MTH TMTQY SM JIOK – SM XPMZ KPEVF ISTTCFKV. ALMK NEK
EB XKEZX I HGKBI VAZMVR WR ZLL KMZKVHP XXGR VJ ZQREFW IZJ
GPVKGOXZ XPMZ LHH TATK ZMVOK KYSEZ VEZX BTK TVMVF CLLVM
MTC ZMVSRI OYUMT GUYTP VSZWQNRC OEDQ G JPVU SXEZT WR ZLL
APARI.

TYTFOZHG EMY WLPN-MJNBWBUTK HRL EKPM-GWDXIJXQZM. MA LIP ZS
II, NAX RVXPUTK OYUMT GUYTP GHQYAF GRK GWDXIJX QF WYPGSXE
IUSCSN SY IDQT EKIYGGXLPG QTSBKP. EU EKITX GRK PCBUZ HXBQTHLH
BTK QVRAFWSBW OUGRA SVXE PPKPFC HRL EATLVNUMHPTK, EIA EA
IKPS EA MTC TIV OUYSH. BTKC MIL UZ HHXI, MJNBWBQJ UBIAFOSUW
BA OXZ RMQJW HRL FXEUWTMZIK XPQ GRZAMDY XOEB IKVL MAEAIK.
GMDZEPRTK ZLLC, IZJ ESP WFNIIY TUQI ALMY, CIYI NGRPF IVFOX SIL FU
WOEZQ OR ALM SRSYC BTGX DEA YAPAMDMI'W.

MSZ PKGHHME, SYSXQHGG OEL TKPWIL PKWPKV FNI ZLQBY EUH XXUX
ALM FXEQIKFUVPIA FNEA IVMHPLH UMT XV VMMIL ALM YUSU, QIDY,
EUH DQTYZ, FCF VEZX BTGX, LEZFN'W WSWD XIZSCDIIZ GWGRH USB
EATWSZF ZLL WPUVW. ASW YAGO IVQXKF AIE TILHMP LSY XPQ RSUK
BDOTZ. IIDZL LBXXUMAIL UZW JSIX GRK YZMTMBQ EUZL PRKDKEZMVS
KJMMKUKRJC, JGZ XOIZQ CEZ SVXE WV QCON SM FWFN.

FBX AXUASC UGRXPZIO RIHVVQJ IUSCSN XV EVECIV HMQVIV
UCQYXPSVE SSIY NGTHHQZZESPG, MTH VR UME 14, 2061, AOEB
TGH IIMZ ZLLSZK, HIJEUQ LEJX.

- (i) For $\tau = 1, 2, \dots, 10$ compute $S_\tau = \sum_{i=0}^{25} q_{\tau,i}^2$ where $q_{\tau,i}$ is the frequency of the letter i in the ciphertext letters $c_1, c_{1+\tau}, c_{1+2\tau}, \dots$. Let τ_0 be the value for which S_τ is closest to the 'English' value $\sum_{i=0}^{25} p_i^2 \approx 0.065$ where p_i is the frequency of letter i in English. 4

- +4** (ii) For each offset $a \in \{0, 1, 2, \dots, \tau_0 - 1\}$ compute the key letter $k_a \in \{0, 1, 2, \dots, 25\}$ for which $\sum_{i=0}^{25} p_i r_{a,i+k_a}$ is closest to 0.065. Here $r_{a,i}$ is the frequency of letter i in $c_a, c_{a+\tau}, c_{a+2\tau}, \dots$
- 4** (iii) Find the key k and decrypt.

Exercise 2.2.

(6 points)

- 6** When using the one-time pad (Vernam's cipher) with the key $k = 0^\kappa$, it follows that $\text{Enc}_k(m) = m$ and the message is effectively sent in the clear. It has therefore been suggested to improve the one-time pad by only encrypting with a key $k \neq 0^\kappa$, ie. to have KeyGen choose k uniformly at random from the set of *non-zero* keys of length κ . Is this an improvement? In particular, is it still perfectly secret? Prove your answer.

If your answer is positive, explain why the one-time pad is not described in this way. If your answer is negative, reconcile this with the fact that encrypting with 0^κ doesn't change the plaintext.